



جزوه باما

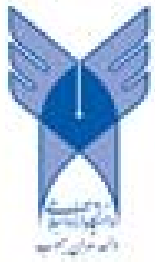
دانلود جزوات، نمونه سوالات
و پروپوزنت‌های دانشگاهی

Jozvebama.ir

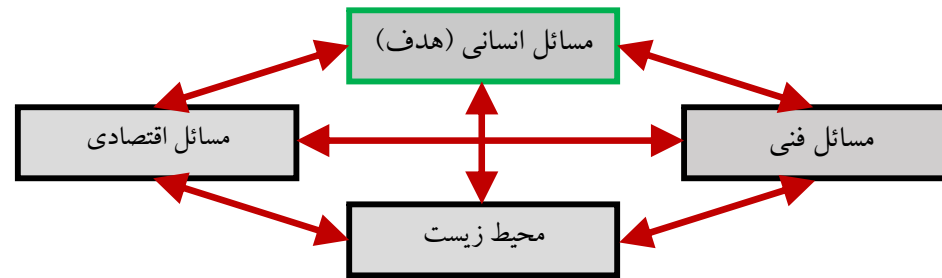


تحلیل سازه ها

کلیات



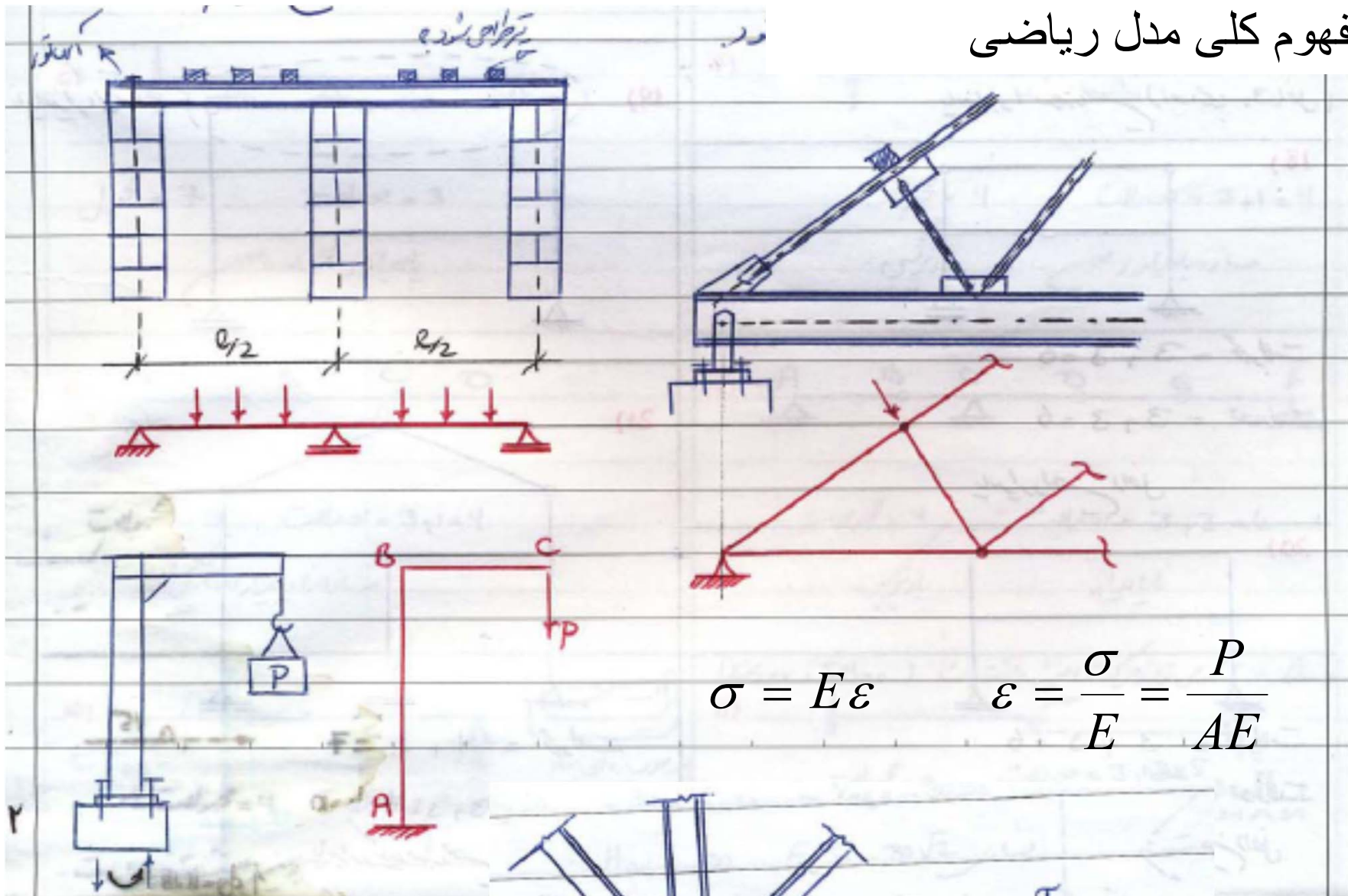
تعریف برخی مفاهیم مهم مهندسی



تعریف مهندسی

مهندسی برقراری یک توازن همه جانبه بین چهار محور "مسائل انسانی"، "محیط زیست"، "مسائل فنی" و "مسائل اقتصادی" است به نحوی که مسائل انسانی در صدر قرار داشته و محور اصلی شمرده می شود.

طراحی تحلیل سازه مدل سازی



$$\sigma = E \varepsilon \quad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$



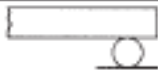



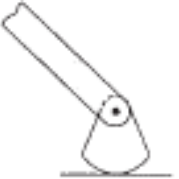

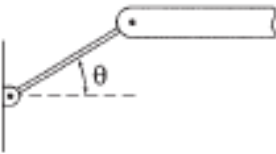

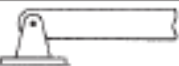

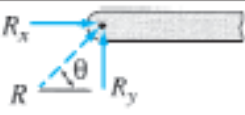
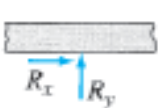
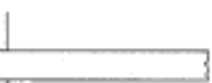
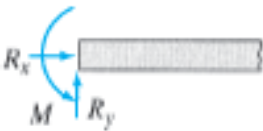
Category	Type of support	Symbolic representation	Reactions	Number of unknowns
I	Roller	 or 	 or 	<p>1</p> <p>The reaction force R acts perpendicular to the supporting surface and may be directed either into or away from the structure. The magnitude of R is the unknown.</p>
	Rocker			<p>1</p> <p>The reaction force R acts in the direction of the link and may be directed either into or away from the structure. The magnitude of R is the unknown.</p>
	Link			<p>1</p> <p>The reaction force R acts in the direction of the link and may be directed either into or away from the structure. The magnitude of R is the unknown.</p>
II	Hinge	 or 	 or 	<p>2</p> <p>The reaction force R may act in any direction. It is usually convenient to represent R by its rectangular components, R_x and R_y. The magnitudes of R_x and R_y are the two unknowns.</p>
III	Fixed			<p>3</p> <p>The reactions consist of two force components R_x and R_y and a couple of moment M. The magnitudes of R_x, R_y, and M are the three unknowns.</p>

FIG. 33 Types of Supports for Plane Structures



معادلات اساسی تحلیل سازه ها

• معادلات تعادل استاتیک

$$\begin{array}{lll} \sum F_x = 0 & \sum F_y = 0 & \sum F_z = 0 \\ \sum M_x = 0 & \sum M_y = 0 & \sum M_z = 0 \end{array}$$

For plane structures, the equations of equilibrium are expressed as

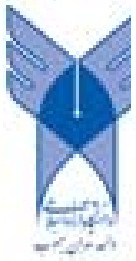
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0$$

• معادلات رفتاری

$$\sigma = E\varepsilon$$

$E =$ Youngs Modulus or
Modulus of Elasticity

• معادلات سازگاری تغییر شکل ها



Stress & Strain: Axial Loading

- رفتار مناسب یک سازه یا ماشین تنها وابسته به نیروها نیست و تغییر شکلها نیز در طراحی سازه نقش دارند
- فرض تغییر شکل پذیری می تواند برای تحلیل سازه های نامعین استاتیکی مورد بهره برداری قرار گیرد
- تعیین توزیع تنش در یک المان سازه ای نیازمند در نظر گرفتن نحوه تغییر مکان سازه است
- بر این اساس در این فصل تغییر شکل یک المان محوری در اثر بارگذاری در امتداد محور این المان مورد بحث قرار می گیرد
- تغییر شکلها در اثر سایر بارگذاریها (خمش، برش، پیچش) در فصل های بعد بررسی می شوند



چرا مفاهیم تنش و کرنش بوجود آمده اند؟

Normal Strain

کرنش قائم

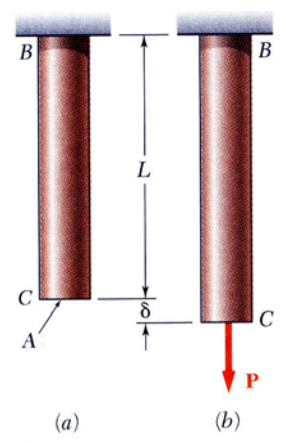


Fig. 2.1

$$\sigma = \frac{P}{A} = \text{stress}$$

$$\epsilon = \frac{\delta}{L} = \text{normal strain}$$

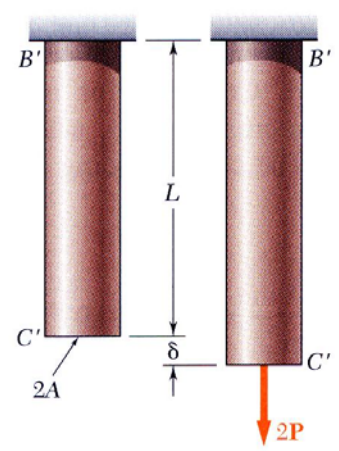


Fig. 2.3

$$\sigma = \frac{2P}{2A} = \frac{P}{A}$$

$$\epsilon = \frac{\delta}{L}$$

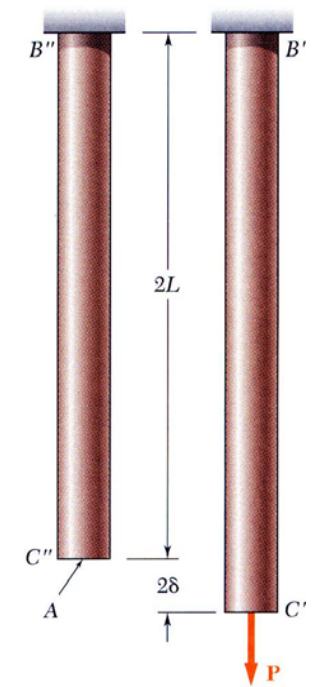


Fig. 2.4

$$\sigma = \frac{P}{A}$$

$$\epsilon = \frac{2\delta}{2L} = \frac{\delta}{L}$$

$$\epsilon = \frac{\delta}{L}$$



آزمایش تنش - کرنش Stress-Strain Test

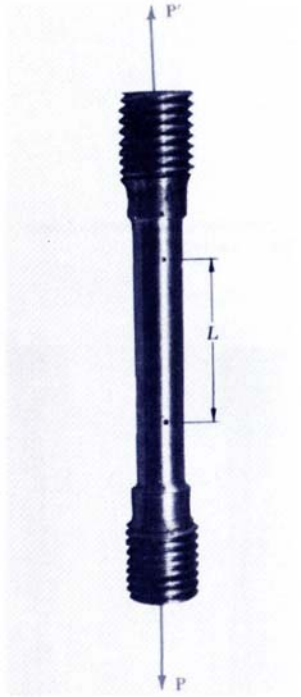


Fig. 2.8 Test specimen with tensile load.

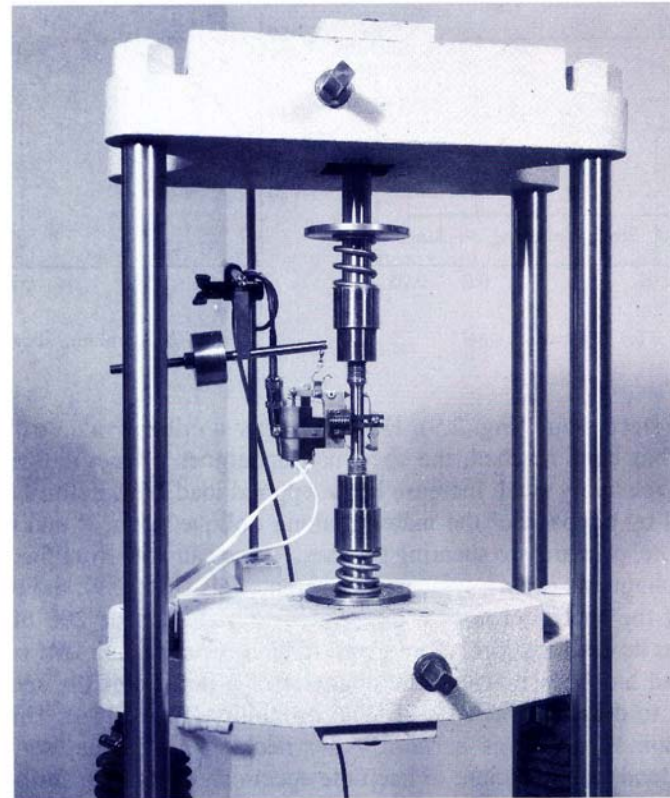
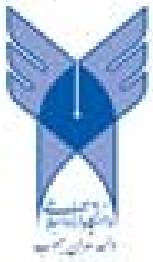
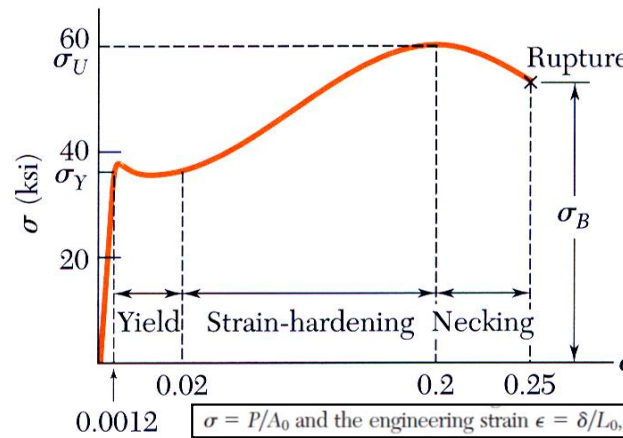
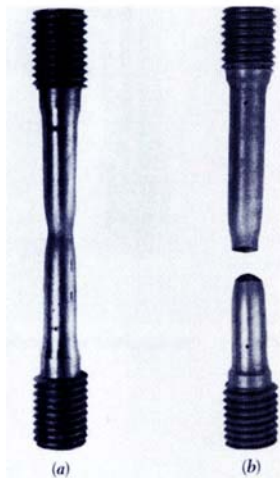


Fig. 2.7 This machine is used to test tensile test specimens, such as those shown in this chapter.

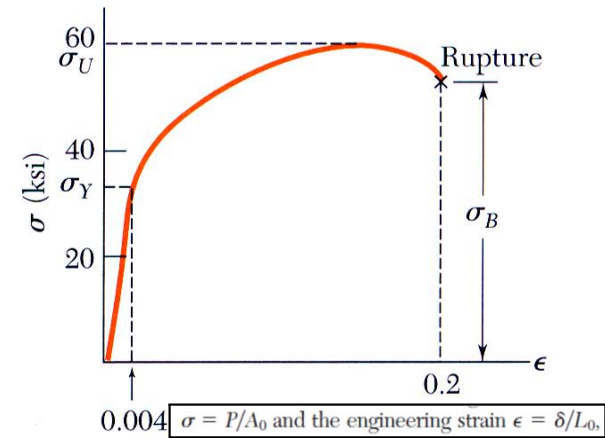


Stress-Strain Diagram: Ductile Materials

نمودار تنش کرنش : ماده شکل پذیر



(a) Low-carbon steel



(b) Aluminum alloy

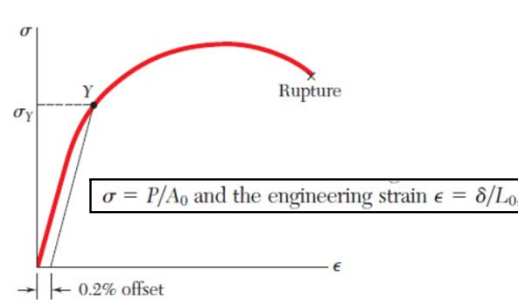


Fig. 2.8 Determination of yield strength by offset method.

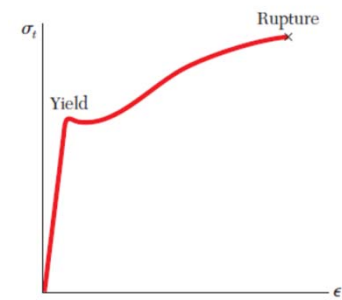
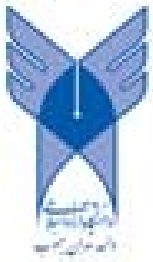


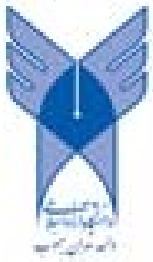
Fig. 2.10 True stress versus true strain for a typical ductile material.



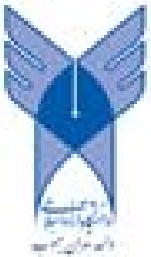
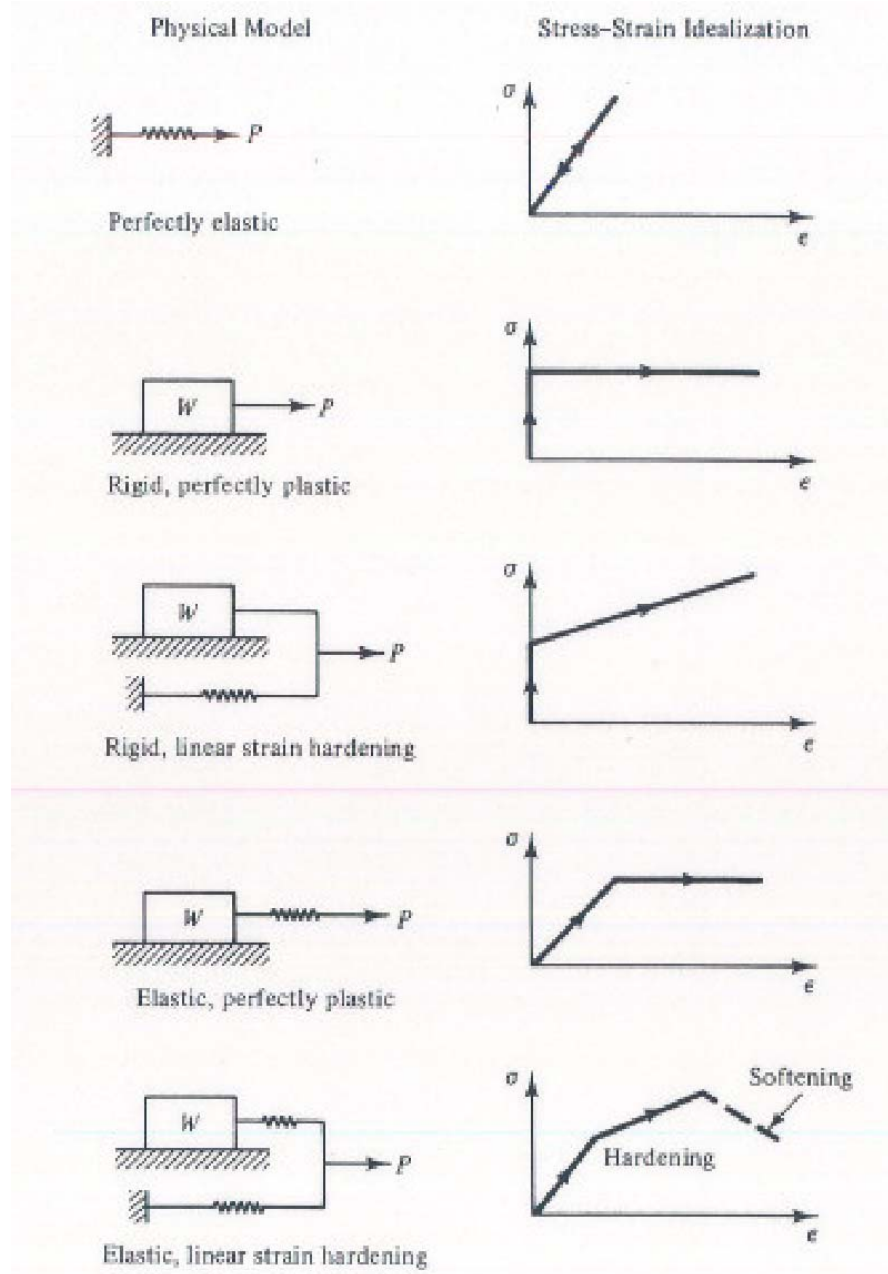
رفتار جسم شکل پذیر

مهمترین ویژگی ذاتی در یک جسم **شکل پذیر** رفتار **تسلیم پذیری** آن است

- تسلیم به معنای آن است که در یک جسم بدون افزایش (قابل ملاحظه) در تنش کرنش های قابل توجهی بوجود آید.
- این تغییر شکل به دلیل لغزش صفحات مایل تحت بر اتفاق می افتد.
- تغییر شکل یک المان بعد از شروع تسلیم می تواند تا ۲۰۰ برابر تغییر شکل در زمان شروع تسلیم باشد.
- غالبا از یک مرحله به بعد در اثر ناپایداری داخلی در المان سطح مقطع (به طور قابل توجهی) کوچک می شود که به این پدیده گردنی شدن (Necking) گفته می شود. زاویه این مخروط ناقص حدود ۴۵ درجه است که حکایت از نقش تنش های برشی دارد.
- در نهایت المان در امتداد این سطح گسیخته می شود.



مدل فنر-لغزنده برای رفتار مواد پس از تسلیم



Stress-Strain Diagram: Brittle Materials

نمودار تنش کرنش : ماده ترد

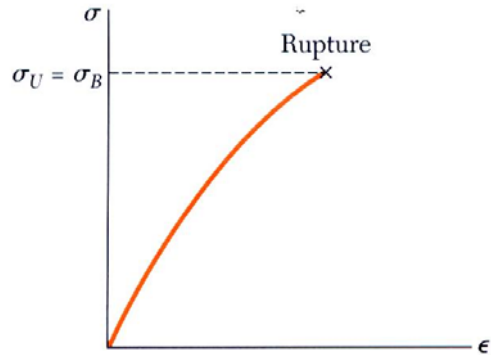


Fig. 2.11 Stress-strain diagram for a typical brittle material.



رفتار جسم ترد

جسم ترد (مثل شیشه ، سنگ ، بتن ، چدن) قبل از تحمل کرنش های نسبی زیاد گسیخته می شود. بنابراین در یک جسم ترد تفاوت کمتری بین تنش نهایی و تنش گسیختگی وجود دارد.

- در یک جسم ترد معمولا شاهد گردنی شدن نیستیم.
- گسیختگی در امتداد سطح عمود بر بارگذاری اتفاق می افتد



قانون هوک و مدول الاستیک

Hooke's Law: Modulus of Elasticity

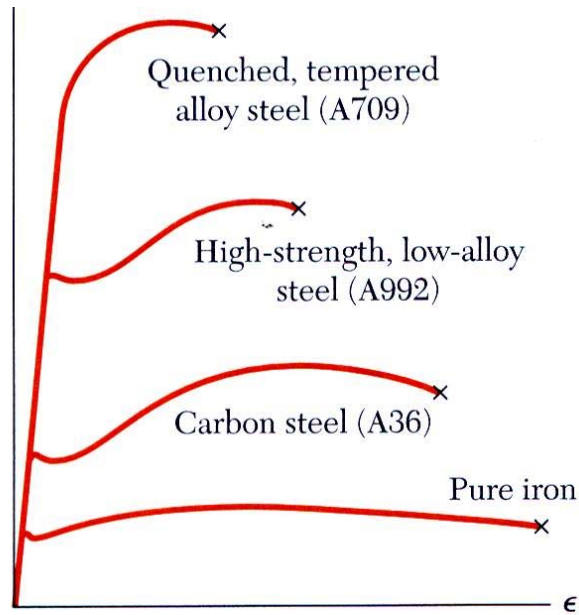


Fig. 2.16 Stress-strain diagrams for iron and different grades of steel.

نگاه مهندسی به فولاد

- در مقادیر تنش کمتر از تسلیم رفتار الاستیک خطی است

$$\sigma = E\varepsilon$$

$E = \text{Young's Modulus or Modulus of Elasticity}$

- مقاومت فولاد تحت تاثیر آلیاژ، عملیات حرارتی و فرآیندهای تولید قرار می گیرد ولی سختی (مدول الاستیک) تاثیری از این موضوعات نمی پذیرد

$$\sigma = E\varepsilon \rightarrow \frac{F}{A} = E \frac{\delta L}{L} \rightarrow F = \frac{EA}{L} \delta L \rightarrow \boxed{k = \frac{EA}{L}}$$

$$F = kx \text{ or } F = k\delta L$$



رفتارهای الاستیک و پلاستیک

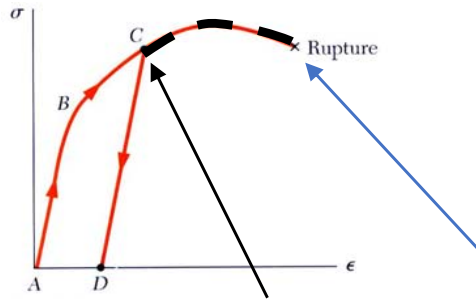


Fig. 2.18 حد تناسب و حد الاستیک

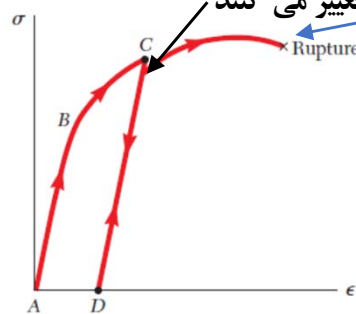


Fig. 2.14 Stress-strain characteristics of ductile material reloaded after prior yielding.

- اگر کرنش ها پس از باربرداری برگردند (صفر شوند) رفتار ماده الاستیک است.
- بیشترین تنشی که رفتار ماده در آن الاستیک باشد را حد الاستیک می نامند.
- اگر کرنش ها پس از باربرداری به مقدار قبل از بارگذاری برگردند (صفر نشوند) رفتار ماده را پلاستیک می نامیم.

تغییر شکل الاستیک = تغییر شکل برگشت پذیر
 تغییر شکل پلاستیک = تغییر شکل ماندگار
 در اکثر مواد مقدار تغییر شکل های پلاستیک فقط به مقدار تنش بستگی ندارد و به سرعت بارگذاری هم وابسته است. بخش وابسته به زمان را لغزش (Slip) این گونه رفتار خزش (Creep) گفته می شود. رفتار های خزشی به دما حساس هستند.
 در مکانیک خاک رفتار وابسته به زمان تحت عنوان تحکیم (Consolidation) شناخته می شوند.



بارگذاری ، باربرداری و بارگذاری معکوس در فولاد

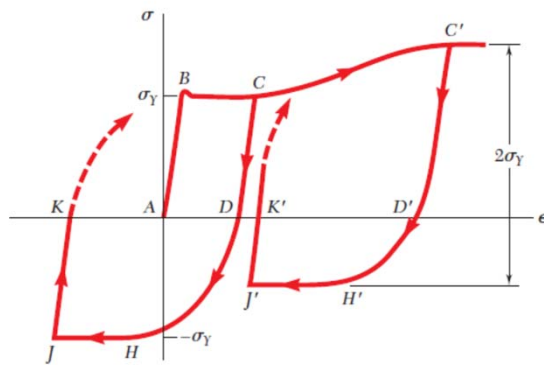
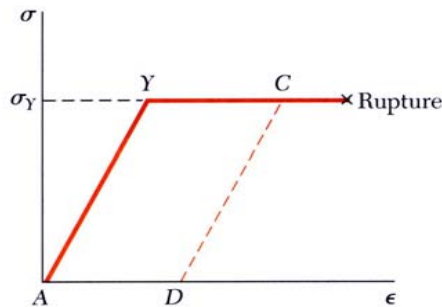


Fig. 2.15 Stress-strain characteristics for mild steel subjected to reverse loading.

- معمولاً در محاسبات متداول مهندسی مصالح را الاستیک خطی فرض می کنند
- این فرض برای مصالح شکل پذیر که سطح تنش در آنها کمتر از تنش تسلیم است مناسب است و همچنین در مصالح ترد که گسیختگی بدون تسلیم رخ می دهد ، قابل کاربرد است.
- در صورتی که سطح تنش افزایش یابد رفتارهای پلاستیک اتفاق می افتد.



- در بسیاری از مسائل واقعی مهندسی برای تحلیل رفتار پلاستیک، از ساده سازی به صورت مقابل استفاده می شود.
- در این ساده سازی، رفتار به صورت رفتار الاستیک-کاملاً پلاستیک مدل می شود.



یک نگاه کوتاه به رفتار تنش - کرنش بتن

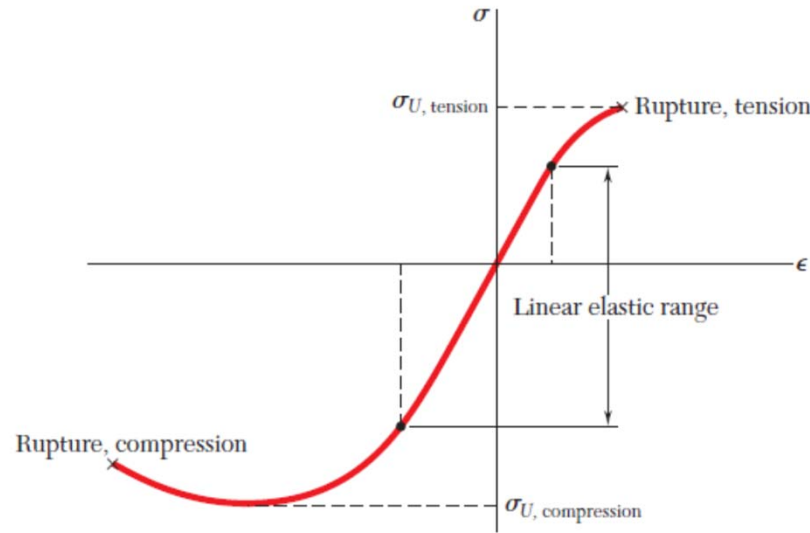
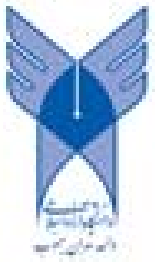
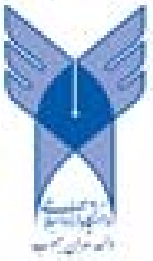
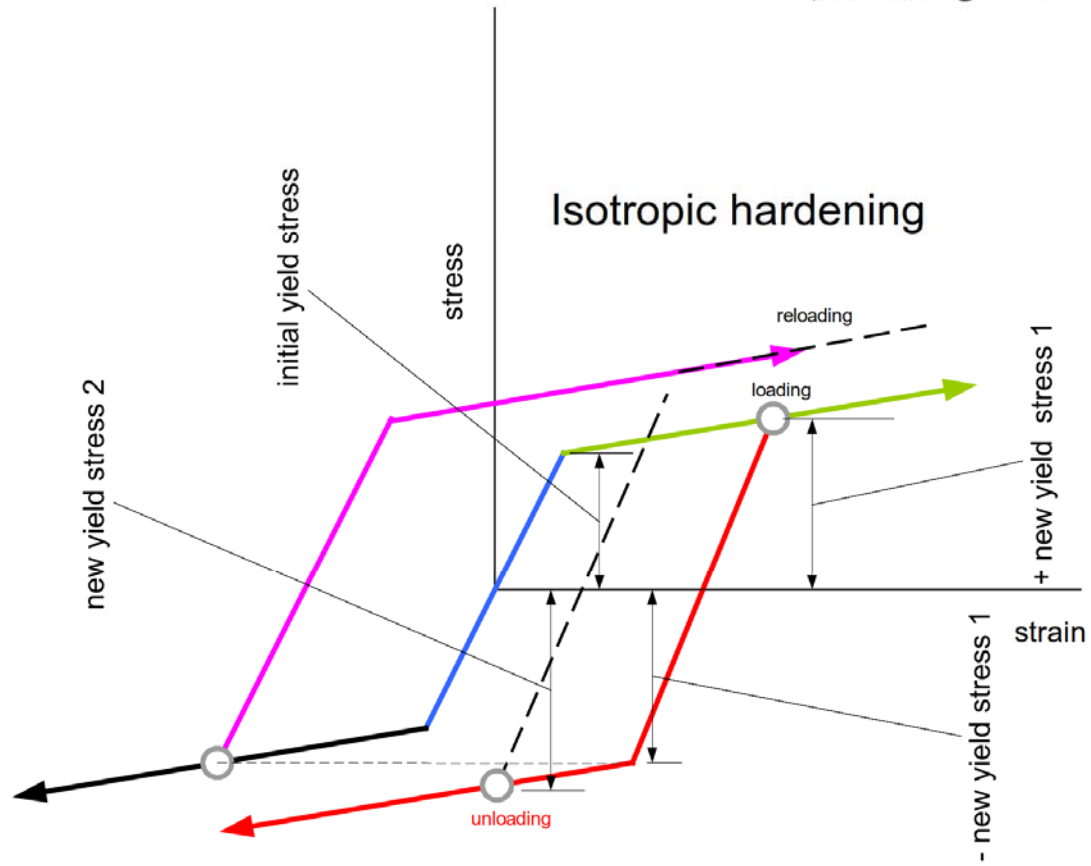


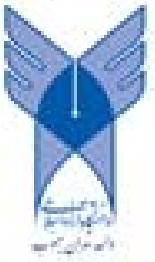
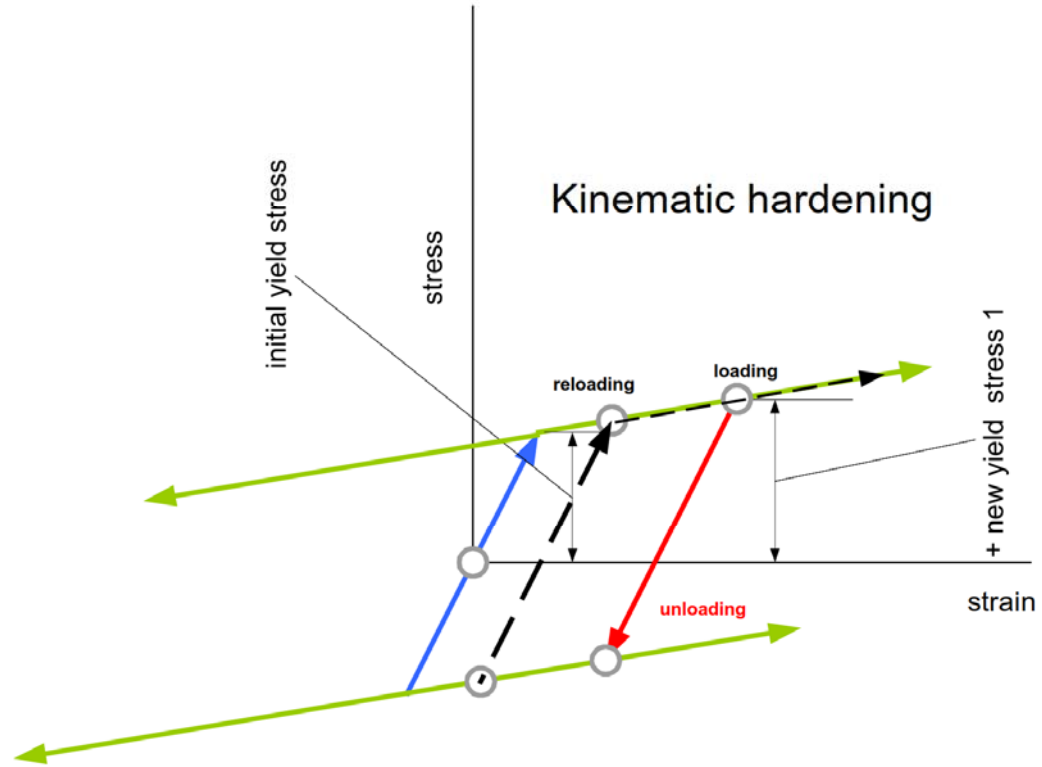
Fig. 2.9 Stress-strain diagram for concrete.



سخت شوندگی ایزوتروپ



سخت شوندگی سینماتیک



خستگی Fatigue

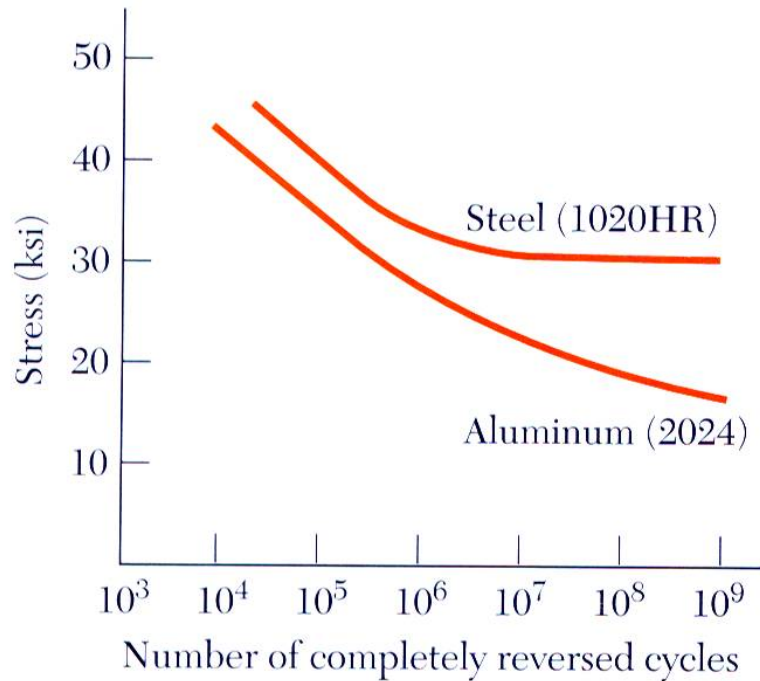
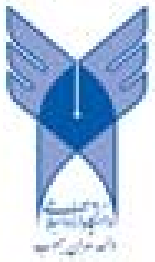


Fig. 2.21

- Fatigue properties are shown on S-N diagrams.
- A member may fail due to *fatigue* at stress levels significantly below the ultimate strength if subjected to many loading cycles.
- When the stress is reduced below the *endurance limit*, fatigue failures do not occur for any number of cycles.



رفتار انیزوتروپیک

برخی از مواد در جهات مختلف رفتارهای متفاوتی دارند ، مثلا مدول الاستیک آن در جهت های گوناگون متفاوت هستند ، به این گونه رفتار ، رفتار آنیزوتروپیک گفته می شود.

مانند: چوب ، بتن مسلح ، فایبر گلس یا کامپوزیت با فیبر های جهت دار

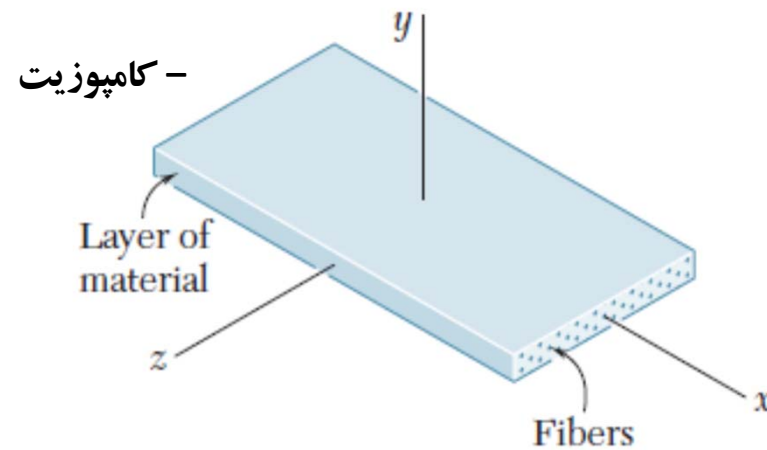


Fig. 2.12 Layer of fiber-reinforced composite material.

معادلات اساسی تحلیل سازه

- معادلات تعادل استاتیکی
- معادلات رفتاری
- معادلات سازگاری تغییر شکل ها

A bar AB of length L and uniform cross section is attached to rigid supports at A and B before being loaded. What are the stresses in portions AC and BC due to the application of a load P at point C (Fig. 2.22a)?

EXAMPLE 2.03

Drawing the free-body diagram of the bar (Fig. 2.22b), we obtain the equilibrium equation

$$R_A + R_B = P \quad (2.14)$$

Since this equation is not sufficient to determine the two unknown reactions R_A and R_B , the problem is statically indeterminate.

However, the reactions may be determined if we observe from the geometry that the total elongation δ of the bar must be zero. Denoting by δ_1 and δ_2 , respectively, the elongations of the portions AC and BC , we write

$$\delta = \delta_1 + \delta_2 = 0$$

or, expressing δ_1 and δ_2 in terms of the corresponding internal forces P_1 and P_2 :

$$\delta = \frac{P_1 L_1}{AE} + \frac{P_2 L_2}{AE} = 0 \quad (2.15)$$

But we note from the free-body diagrams shown respectively in parts b and c of Fig. 2.23 that $P_1 = R_A$ and $P_2 = -R_B$. Carrying these values into (2.15), we write

$$R_A L_1 - R_B L_2 = 0 \quad (2.16)$$

Equations (2.14) and (2.16) can be solved simultaneously for R_A and R_B ; we obtain $R_A = PL_2/L$ and $R_B = PL_1/L$. The desired stresses σ_1 in AC and σ_2 in BC are obtained by dividing, respectively, $P_1 = R_A$ and $P_2 = -R_B$ by the cross-sectional area of the bar:

$$\sigma_1 = \frac{PL_2}{AL} \quad \sigma_2 = -\frac{PL_1}{AL}$$

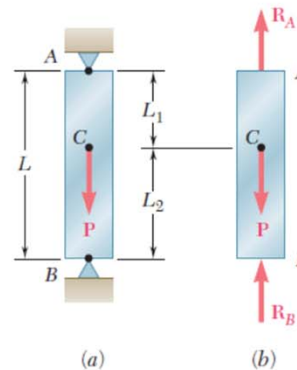


Fig. 2.22

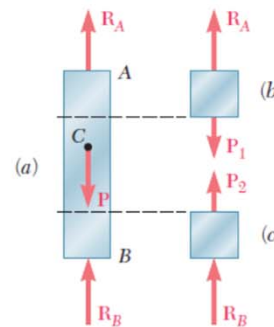
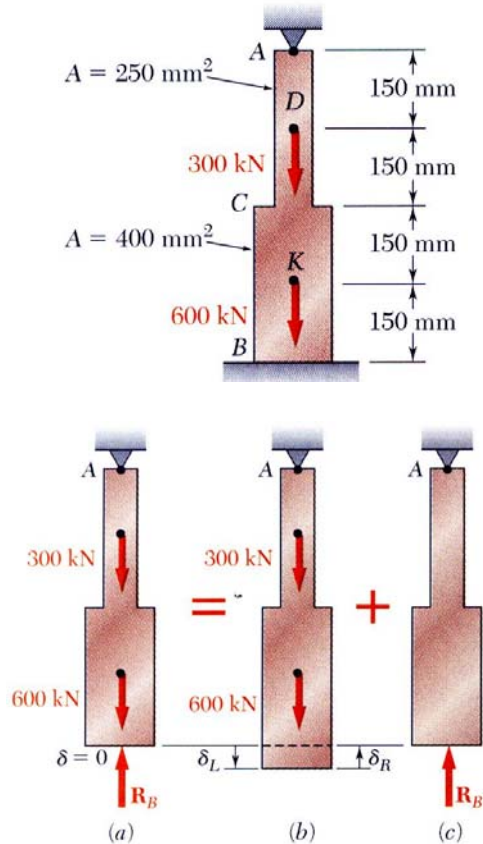


Fig. 2.23

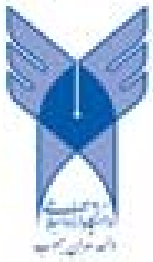


- روش نرمی
- روش سختی



- Structures for which internal forces and reactions cannot be determined from statics alone are said to be *statically indeterminate*.
- A structure will be statically indeterminate whenever it is held by more supports than are required to maintain its equilibrium.
- Redundant reactions are replaced with unknown loads which along with the other loads must produce compatible deformations.
- Deformations due to actual loads and redundant reactions are determined separately and then added or *superposed*.

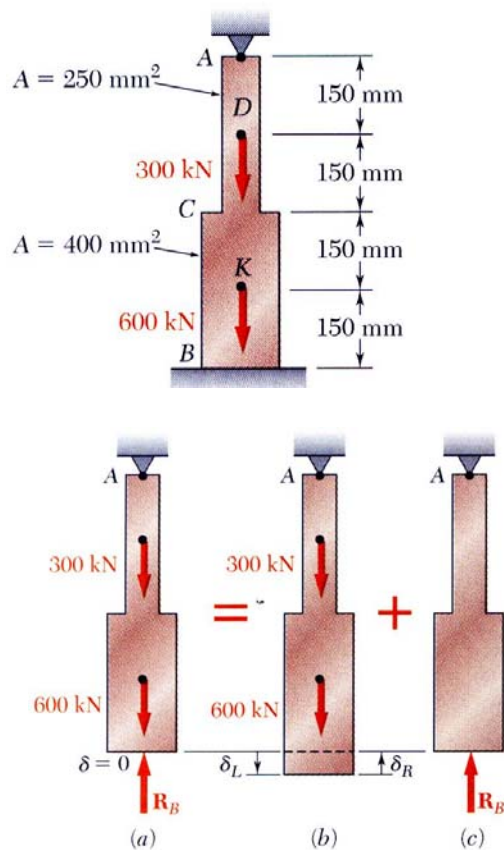
$$\delta = \delta_L + \delta_R = 0$$



Superposition Method. We observe that a structure is statically indeterminate whenever it is held by more supports than are required to maintain its equilibrium. This results in more unknown reactions than available equilibrium equations. It is often found convenient to designate one of the reactions as *redundant* and to eliminate the corresponding support. Since the stated conditions of the problem cannot be arbitrarily changed, the redundant reaction must be maintained in the solution. But it will be treated as an *unknown load* that, together with the other loads, must produce deformations that are compatible with the original constraints. The actual solution of the problem is carried out by considering separately the deformations caused by the given loads and by the redundant reaction, and by adding—or *superposing*—the results obtained. †



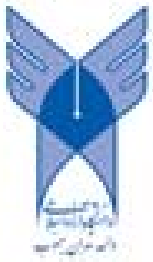
Example 2.04



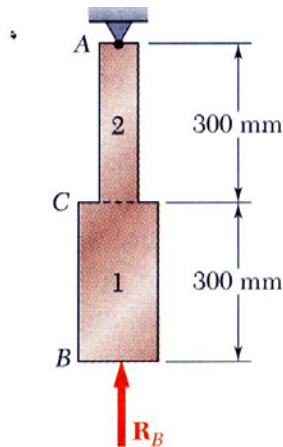
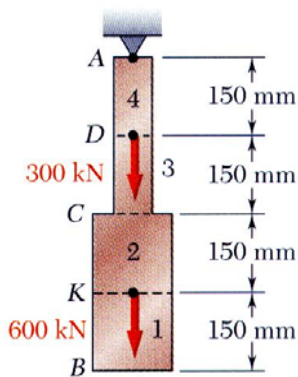
Determine the reactions at A and B for the steel bar and loading shown, assuming a close fit at both supports before the loads are applied.

SOLUTION:

- Consider the reaction at B as redundant, release the bar from that support, and solve for the displacement at B due to the applied loads.
- Solve for the displacement at B due to the redundant reaction at B .
- Require that the displacements due to the loads and due to the redundant reaction be compatible, i.e., require that their sum be zero.
- Solve for the reaction at A due to applied loads and the reaction found at B .



Example 2.04



SOLUTION:

- Solve for the displacement at B due to the applied loads with the redundant constraint released,

$$P_1 = 0 \quad P_2 = P_3 = 600 \times 10^3 \text{ N} \quad P_4 = 900 \times 10^3 \text{ N}$$

$$A_1 = A_2 = 400 \times 10^{-6} \text{ m}^2 \quad A_3 = A_4 = 250 \times 10^{-6} \text{ m}^2$$

$$L_1 = L_2 = L_3 = L_4 = 0.150 \text{ m}$$

$$\delta_L = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1.125 \times 10^9}{E}$$

- Solve for the displacement at B due to the redundant constraint,

$$P_1 = P_2 = -R_B$$

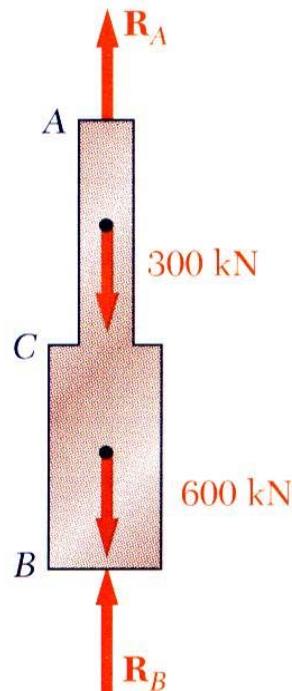
$$A_1 = 400 \times 10^{-6} \text{ m}^2 \quad A_2 = 250 \times 10^{-6} \text{ m}^2$$

$$L_1 = L_2 = 0.300 \text{ m}$$

$$\delta_R = \sum_i \frac{P_i L_i}{A_i E_i} = -\frac{(1.95 \times 10^3) R_B}{E}$$



Example 2.04



- Require that the displacements due to the loads and due to the redundant reaction be compatible,

$$\delta = \delta_L + \delta_R = 0$$

$$\delta = \frac{1.125 \times 10^9}{E} - \frac{(1.95 \times 10^3) R_B}{E} = 0$$

$$R_B = 577 \times 10^3 \text{ N} = 577 \text{ kN}$$

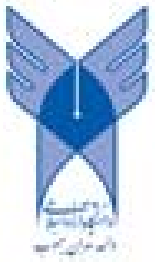
- Find the reaction at A due to the loads and the reaction at

$$\sum F_y = 0 = R_A - 300 \text{ kN} - 600 \text{ kN} + 577 \text{ kN}$$

$$R_A = 323 \text{ kN}$$

$$R_A = 323 \text{ kN}$$

$$R_B = 577 \text{ kN}$$



Deformations Under Axial Loading

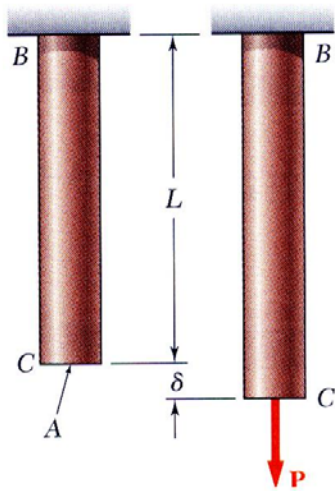


Fig 2.17

- From Hooke's Law:

$$\sigma = E \epsilon \quad \epsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

- From the definition of strain:

$$\epsilon = \frac{\delta}{L}$$

- Equating and solving for the deformation,

$$\delta = \frac{PL}{AE}$$

- With variations in loading, cross-section or material properties,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

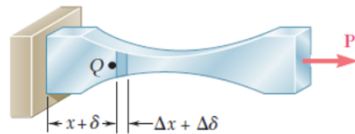
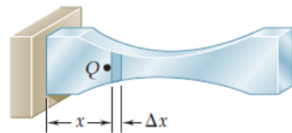
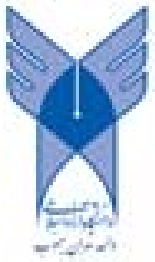


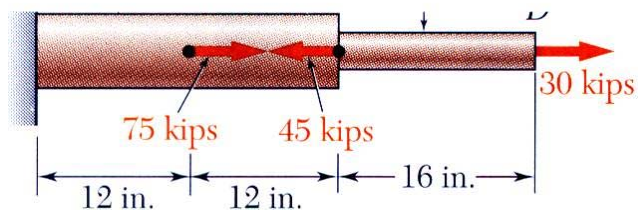
Fig. 2.18 Deformation of axially loaded member of variable cross-sectional area.

$$d\delta = \epsilon dx = \frac{P dx}{AE}$$

$$\delta = \int_0^L \frac{P dx}{AE}$$



Example 2.01



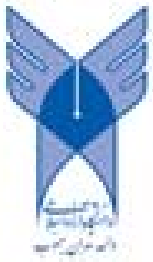
$$E = 29 \times 10^6 \text{ psi}$$

$$D = 1.07 \text{ in.} \quad d = 0.618 \text{ in.}$$

Determine the deformation of the steel rod shown under the given loads.

SOLUTION:

- Divide the rod into components at the load application points.
- Apply a free-body analysis on each component to determine the internal force
- Evaluate the total of the component deflections.



SOLUTION:

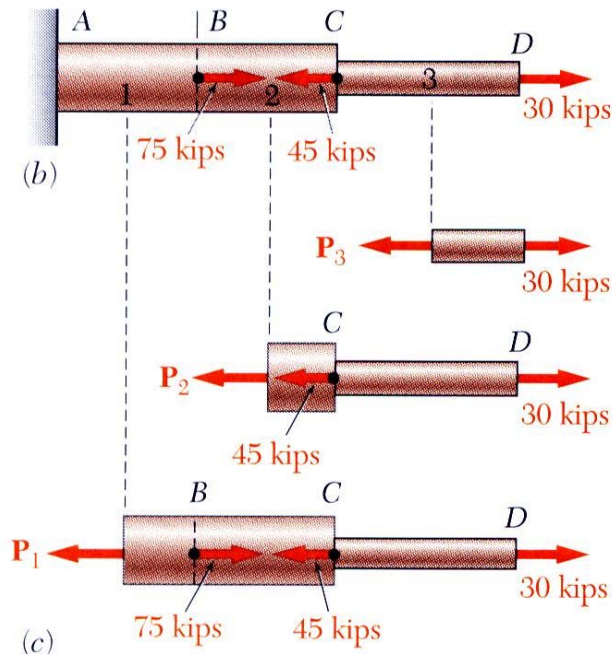
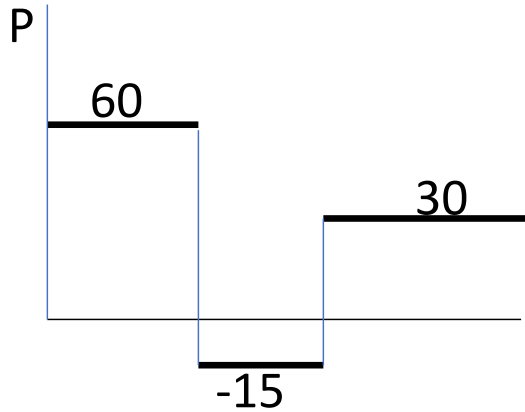
- Divide the rod into three components:

$$L_1 = L_2 = 12 \text{ in.}$$

$$A_1 = A_2 = 0.9 \text{ in}^2$$

$$L_3 = 16 \text{ in.}$$

$$A_3 = 0.3 \text{ in}^2$$



- Apply free-body analysis to each component to determine internal forces,

$$P_1 = 60 \times 10^3 \text{ lb}$$

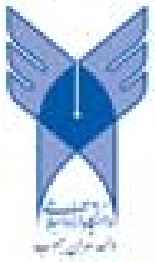
$$P_2 = -15 \times 10^3 \text{ lb}$$

$$P_3 = 30 \times 10^3 \text{ lb}$$

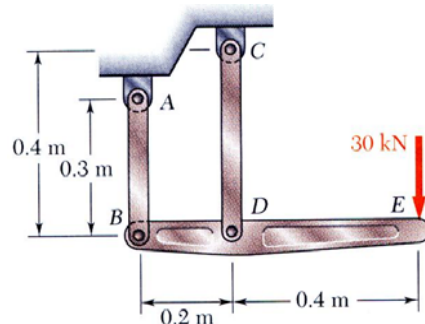
- Evaluate total deflection,

$$\begin{aligned} \delta &= \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1}{E} \left(\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3} \right) \\ &= \frac{1}{29 \times 10^6} \left[\frac{(60 \times 10^3) 12}{0.9} + \frac{(-15 \times 10^3) 12}{0.9} + \frac{(30 \times 10^3) 16}{0.3} \right] \\ &= 75.9 \times 10^{-3} \text{ in.} \end{aligned}$$

$$\delta = 75.9 \times 10^{-3} \text{ in.}$$



Sample Problem 2.1



The rigid bar BDE is supported by two links AB and CD .

Link AB is made of aluminum ($E = 70$ GPa) and has a cross-sectional area of 500 mm². Link CD is made of steel ($E = 200$ GPa) and has a cross-sectional area of (600 mm²).

For the 30-kN force shown, determine the deflection a) of B , b) of D , and c) of E .

SOLUTION:

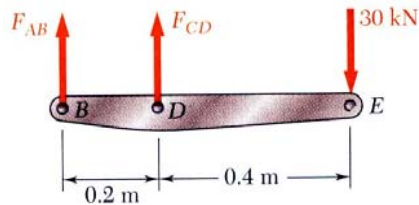
- Apply a free-body analysis to the bar BDE to find the forces exerted by links AB and CD .
- Evaluate the deformation of links AB and CD or the displacements of B and D .
- Work out the geometry to find the deflection at E given the deflections at B and D .



Sample Problem 2.1

SOLUTION:

Free body: Bar *BDE*



$$\sum M_B = 0$$

$$0 = -(30 \text{ kN} \times 0.6 \text{ m}) + F_{CD} \times 0.2 \text{ m}$$

$$F_{CD} = +90 \text{ kN } \textit{tension}$$

$$\sum M_D = 0$$

$$0 = -(30 \text{ kN} \times 0.4 \text{ m}) - F_{AB} \times 0.2 \text{ m}$$

$$F_{AB} = -60 \text{ kN } \textit{compression}$$

Displacement of *B*:

$$\delta_B = \frac{PL}{AE}$$

$$= \frac{(-60 \times 10^3 \text{ N})(0.3 \text{ m})}{(500 \times 10^{-6} \text{ m}^2)(70 \times 10^9 \text{ Pa})}$$

$$= -514 \times 10^{-6} \text{ m}$$

$$\delta_B = 0.514 \text{ mm } \uparrow$$

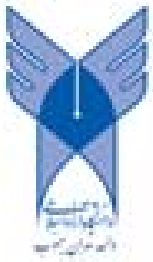
Displacement of *D*:

$$\delta_D = \frac{PL}{AE}$$

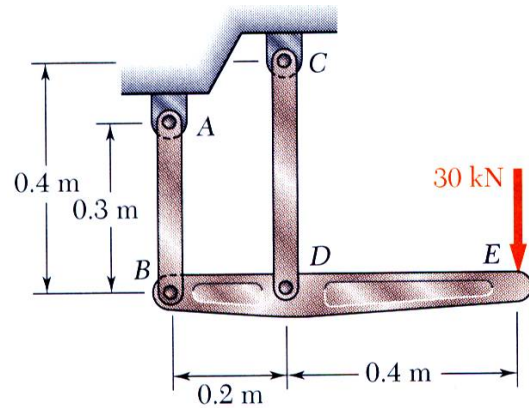
$$= \frac{(90 \times 10^3 \text{ N})(0.4 \text{ m})}{(600 \times 10^{-6} \text{ m}^2)(200 \times 10^9 \text{ Pa})}$$

$$= 300 \times 10^{-6} \text{ m}$$

$$\delta_D = 0.300 \text{ mm } \downarrow$$



Sample Problem 2.1

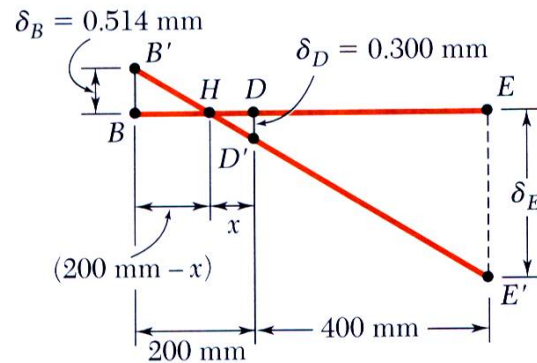


Displacement of D:

$$\frac{BB'}{DD'} = \frac{BH}{HD}$$

$$\frac{0.514 \text{ mm}}{0.300 \text{ mm}} = \frac{(200 \text{ mm}) - x}{x}$$

$$x = 73.7 \text{ mm}$$



$$\frac{EE'}{DD'} = \frac{HE}{HD}$$

$$\frac{\delta_E}{0.300 \text{ mm}} = \frac{(400 + 73.7) \text{ mm}}{73.7 \text{ mm}}$$

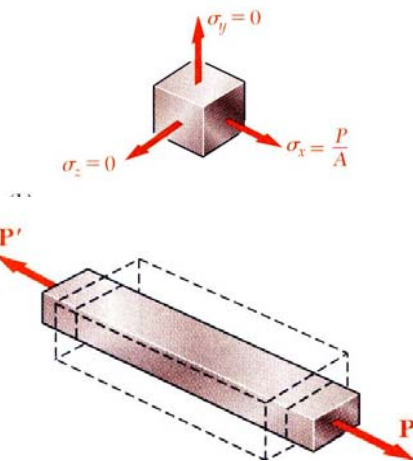
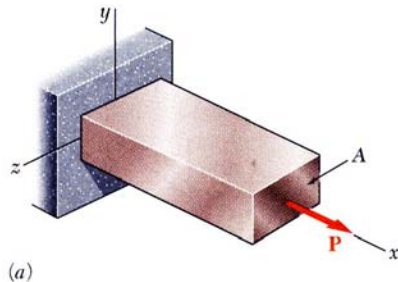
$$\delta_E = 1.928 \text{ mm}$$

$$\delta_E = 1.928 \text{ mm} \downarrow$$



اثر پواسون - ضریب پواسون

Poisson's Ratio



- For a slender bar subjected to axial loading:

$$\epsilon_x = \frac{\sigma_x}{E} \quad \sigma_y = \sigma_z = 0$$

- The elongation in the x-direction is accompanied by a contraction in the other directions. Assuming that the material is isotropic (no directional dependence),

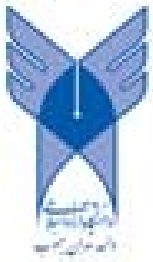
$$\epsilon_y = \epsilon_z \neq 0$$

- Poisson's ratio is defined as

$$\nu = \left| \frac{\text{lateral strain}}{\text{axial strain}} \right| = -\frac{\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x}$$

$$\epsilon_x = \frac{\sigma_x}{E}$$

$$\epsilon_y = \epsilon_z = -\frac{\nu\sigma_x}{E}$$



کرنش برشی Shearing Strain

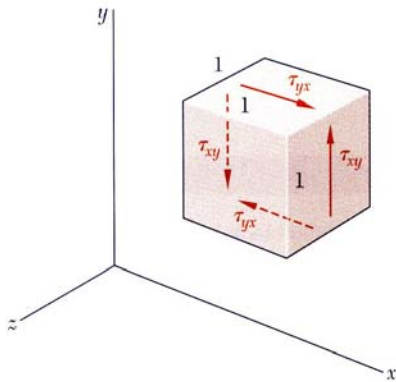


Fig. 2.46

- A cubic element subjected to a shear stress will deform into a rhomboid. The corresponding *shear* strain is quantified in terms of the change in angle between the sides,

$$\tau_{xy} = f(\gamma_{xy})$$

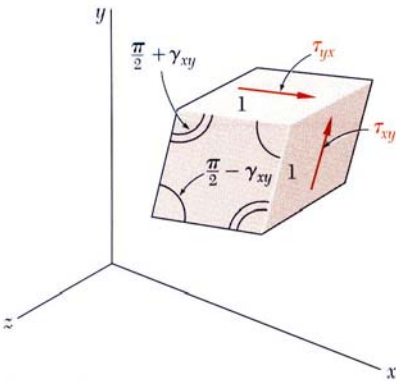
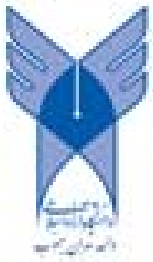


Fig. 2.47

- A plot of shear stress vs. shear strain is similar the previous plots of normal stress vs. normal strain except that the strength values are approximately half. For small strains,

$$\tau_{xy} = G \gamma_{xy} \quad \tau_{yz} = G \gamma_{yz} \quad \tau_{zx} = G \gamma_{zx}$$

where G is the modulus of rigidity or shear modulus.



$$\tau_{xy} = G \gamma_{xy}$$

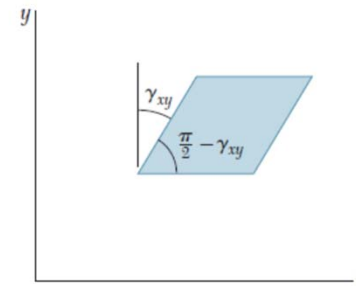


Fig. 2.44

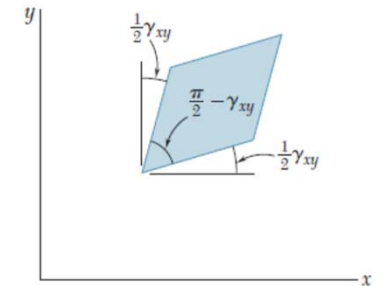
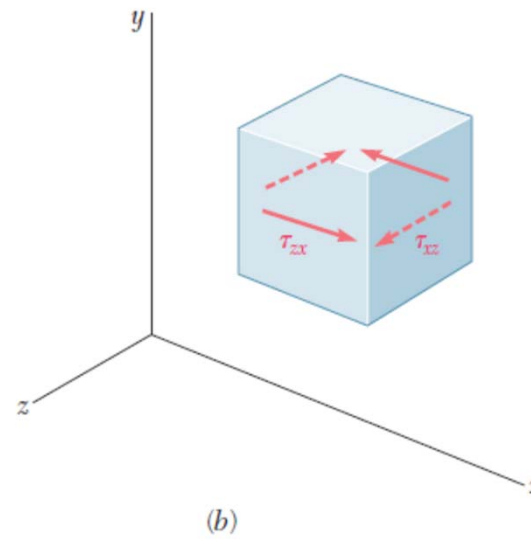
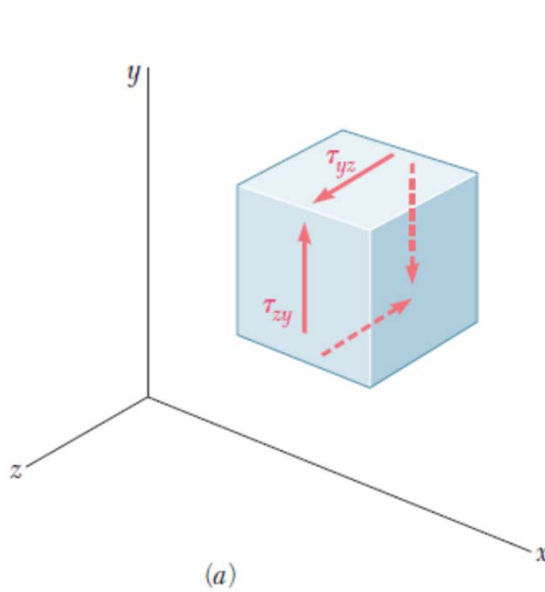


Fig. 2.45

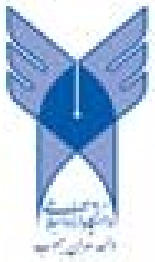
گرنش برشی Shearing Strain



$$\tau_{xy} = G\gamma_{xy}$$

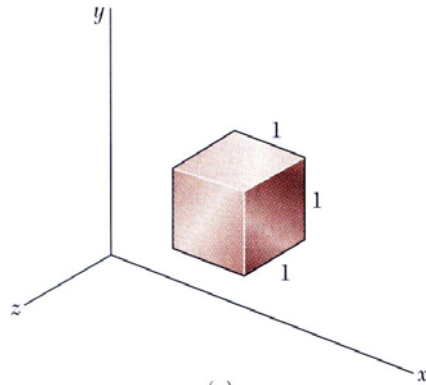
$$\tau_{yz} = G\gamma_{yz}$$

$$\tau_{zx} = G\gamma_{zx}$$

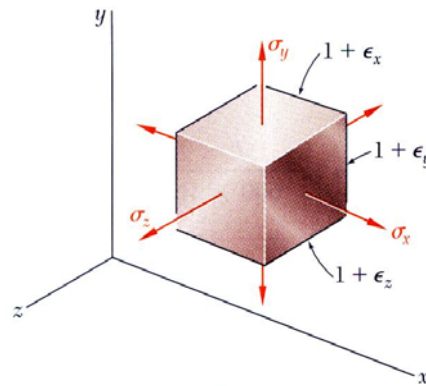


تعمیم قانون هوک

Generalized Hooke's Law



(a)



(b)

- For an element subjected to multi-axial loading, the normal strain components resulting from the stress components may be determined from the *principle of superposition*. This requires:

- 1) strain is linearly related to stress
- 2) deformations are small

- With these restrictions:

$$\epsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\epsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

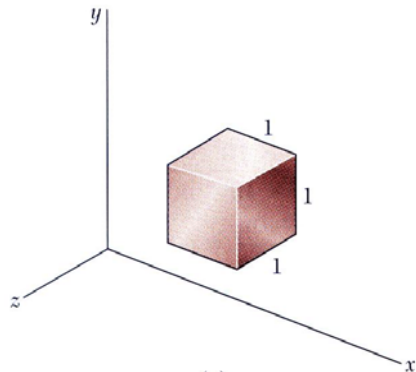
$$\epsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \quad \gamma_{yz} = \frac{\tau_{yz}}{G} \quad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

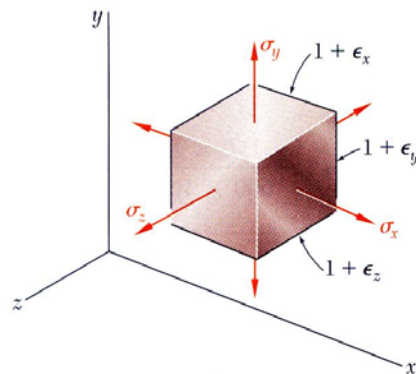


رفتار های حجمی

Dilatation/Compression: Bulk Modulus



(a)



(b)

- Relative to the unstressed state, the change in volume is

$$e = 1 - [(1 + \epsilon_x)(1 + \epsilon_y)(1 + \epsilon_z)] = 1 - [1 + \epsilon_x + \epsilon_y + \epsilon_z]$$

$$= \epsilon_x + \epsilon_y + \epsilon_z$$

$$= \frac{1 - 2\nu}{E} (\sigma_x + \sigma_y + \sigma_z)$$
 = dilatation (change in volume per unit volume)

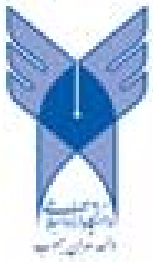
- For element subjected to uniform hydrostatic pressure,

$$e = -p \frac{3(1 - 2\nu)}{E} = -\frac{p}{k}$$

$$k = \frac{E}{3(1 - 2\nu)} = \text{bulk modulus}$$

- Subjected to uniform pressure, dilatation must be negative, therefore

$$0 < \nu < \frac{1}{2}$$





جزوه باما

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