تحلیل تیرهای معین

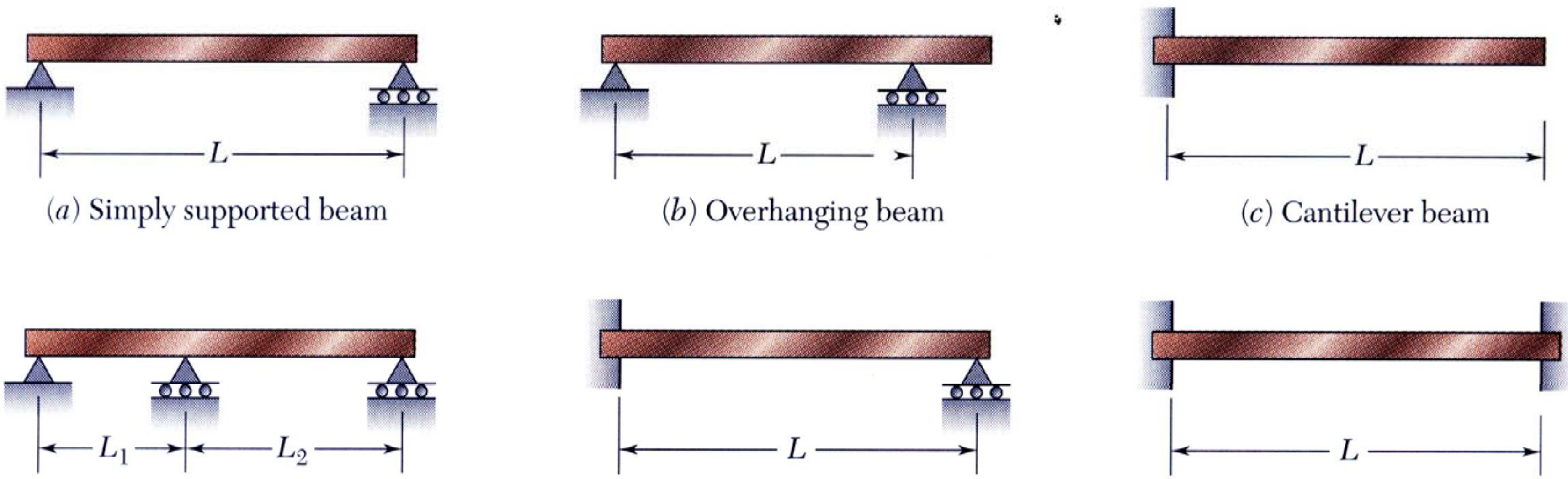
رسم دیاگرام نیروهای داخلی

Introduction

Classification of Beam Supports

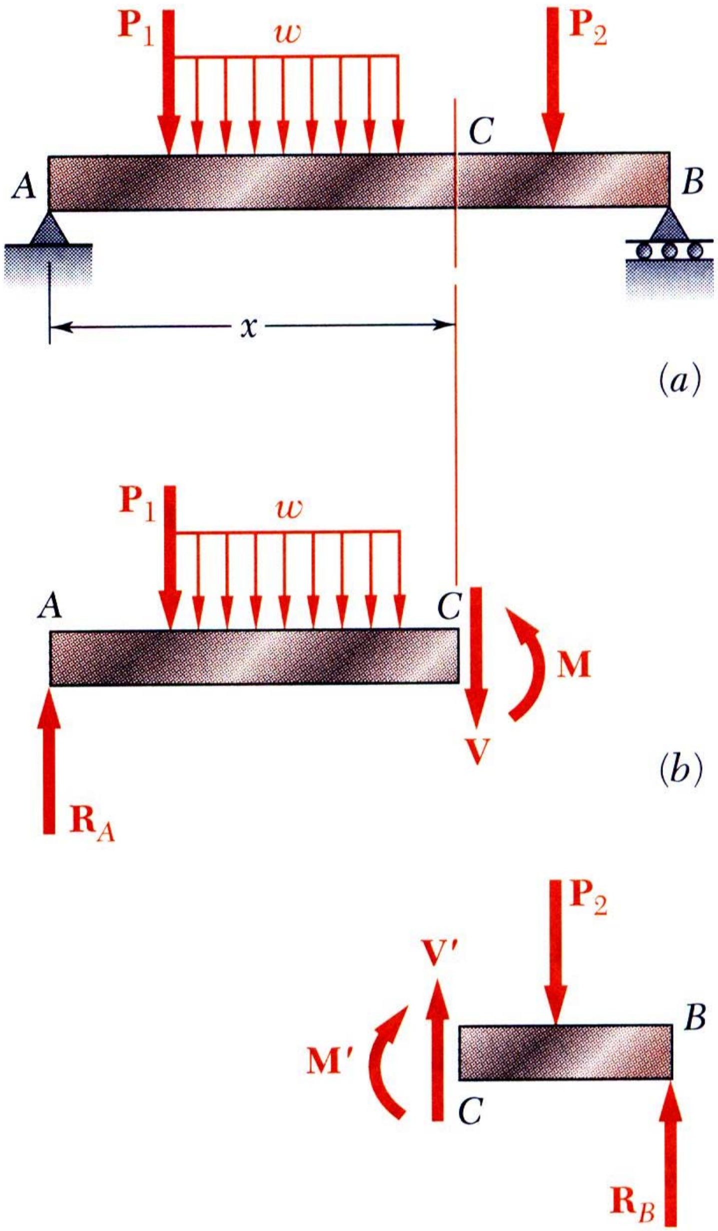
Statically Determinate Beams

Statically Indeterminate Beams



(*a*) Simply supported beam
(*b*) Overhanging beam
(*c*) Cantilever beam
(*d*) Continuous beam
(*e*) Beam fixed at one end and simply supported at the other end
(*f*) Fixed beam

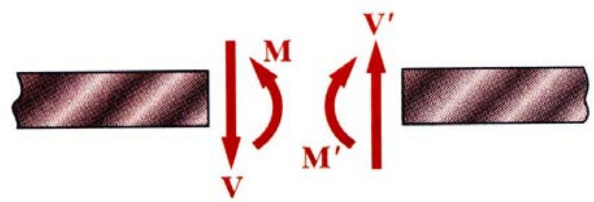
Shear and Bending Moment Diagrams



- Determination of maximum normal and shearing stresses requires identification of maximum internal shear force and bending couple.

- Shear force and bending couple at a point are determined by passing a section through the beam and applying an equilibrium analysis on the beam portions on either side of the section.

- Sign conventions for shear forces *V* and *V'* and bending couples *M* and *M'*



(*a*) Internal forces
(positive shear and positive bending moment)

روش گام به گام رسم دیاگرام های برش و خمش در تیر ها

قواعد کلی :

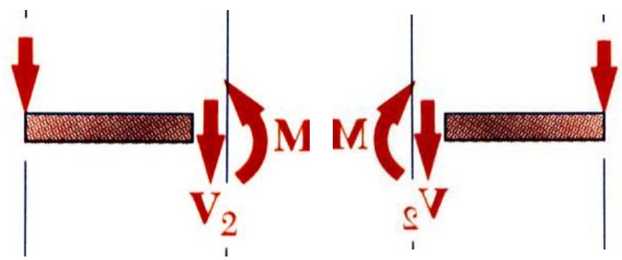
۱- جهت مثبت نیروهای داخلی تیر به صورت مقابل است

۲- سطح زیر نمودار بار برابر است با تغییرات برش $\Delta V = A_q$

۳- سطح زیر نمودار برش برابر است با تغییرات لنگر $\Delta M = A_V$

تذکر : در صورت وجود بار P (لنگر M) متمرکز، مقدار بار P (لنگر M) متمرکز عملا سطح زیر نمودار محسوب می شوند .

$$\Delta V = P$$
$$\Delta M = M$$



رضا عباسی- دانشگاه آزاد اسلامی واحد تهران جنوب

روش گام به گام رسم دیاگرام های برش و خمش در تیر ها

برای رسم دیاگرام برش (خمش):

۱- از سمت چپ به راست تیر حرکت می کنیم.

۲- برش (خمش) در نقطه دوم برابر است با برش (خمش) در نقطه اول باضافه سطح زیر نمودار بار(برش) بین دو نقطه

$$V_2 = V_2 + \Delta V \qquad ; \Delta V = A_q \quad or \quad \Delta V = P$$

$$M_2 = M_2 + \Delta M \qquad ; \Delta M = A_V \quad or \quad \Delta M = M$$

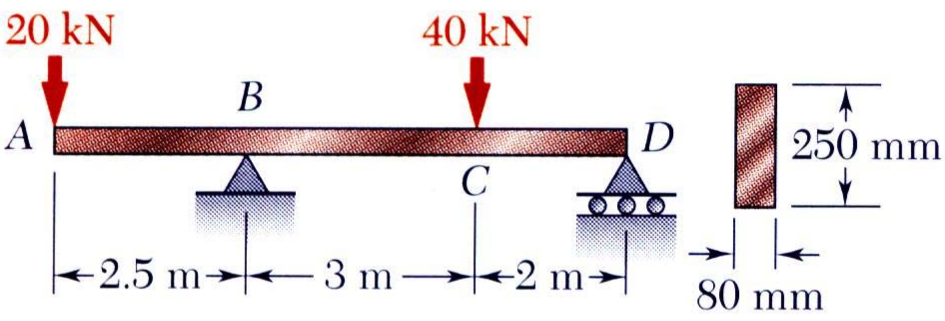
تذکر ۱ : واضح است که جهت نیروهای خارجی برعکس جهت نیروهای داخلی است (قانون سوم نیوتن)

- بار خارجی رو به بالا مثبت
- لنگر خارجی در جهت عقربه های ساعت مثبت

تذکر ۲ : واضح است که اگر منحنی بار از درجه n باشد ، منحنی برش از درجه n+1 و منحنی خمش از درجه n+2 است.

تذکر ۳ : واضح است که هر جا بار (برش) صفرباشد ، برش(لنگر) اکسترمم است.

Sample Problem 5.1

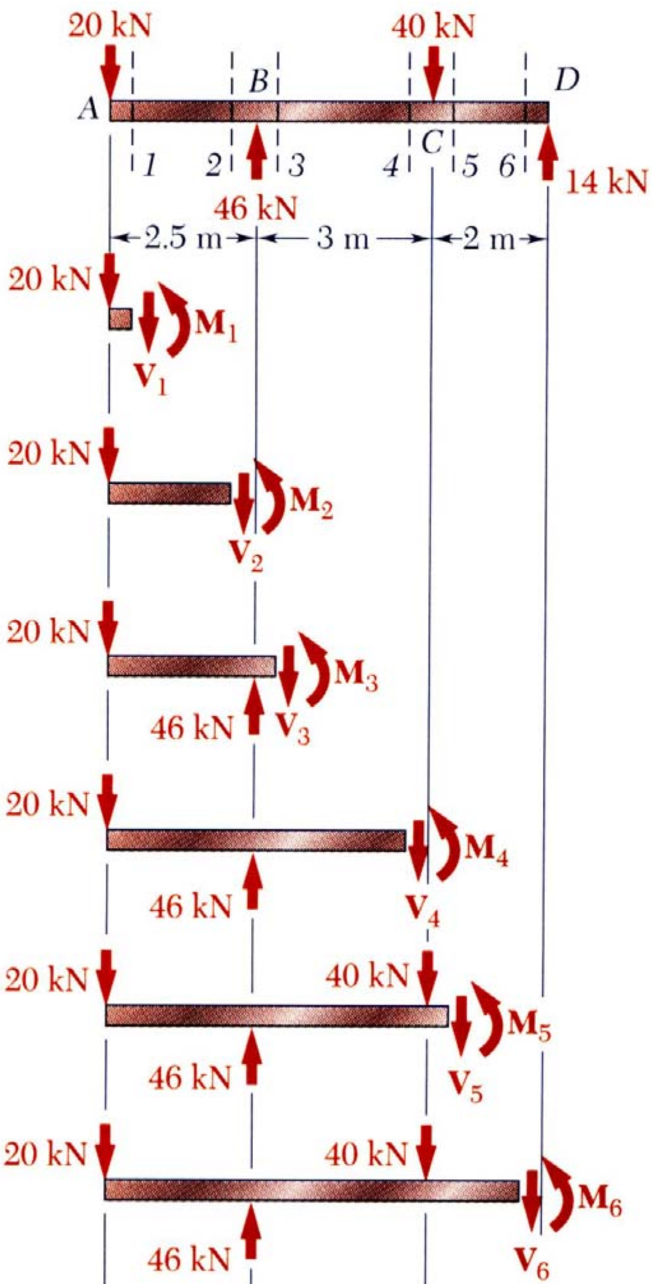


For the timber beam and loading shown, draw the shear and bend-moment diagrams and determine the maximum normal stress due to bending.

SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces
- Section the beam at points near supports and load application points. Apply equilibrium analyses on resulting free-bodies to determine internal shear forces and bending couples
- Identify the maximum shear and bending-moment from plots of their distributions.
- Apply the elastic flexure formulas to determine the corresponding maximum normal stress.

Sample Problem 5.1



SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces

  from $\sum F_y = 0 = \sum M_B : \quad R_B = 40\,\text{kN} \quad R_D = 14\,\text{kN}$

- Section the beam and apply equilibrium analyses on resulting free-bodies

$$\sum F_y = 0 \quad -20\text{ kN} - V_1 = 0 \qquad V_1 = -20\,\text{kN}$$

$$\sum M_1 = 0 \quad (20\,\text{kN})(0\,\text{m}) + M_1 = 0 \quad M_1 = 0$$

$$\sum F_y = 0 \quad -20\text{ kN} - V_2 = 0 \qquad V_2 = -20\,\text{kN}$$

$$\sum M_2 = 0 \quad (20\,\text{kN})(2.5\,\text{m}) + M_2 = 0 \quad M_2 = -50\,\text{kN}\cdot\text{m}$$

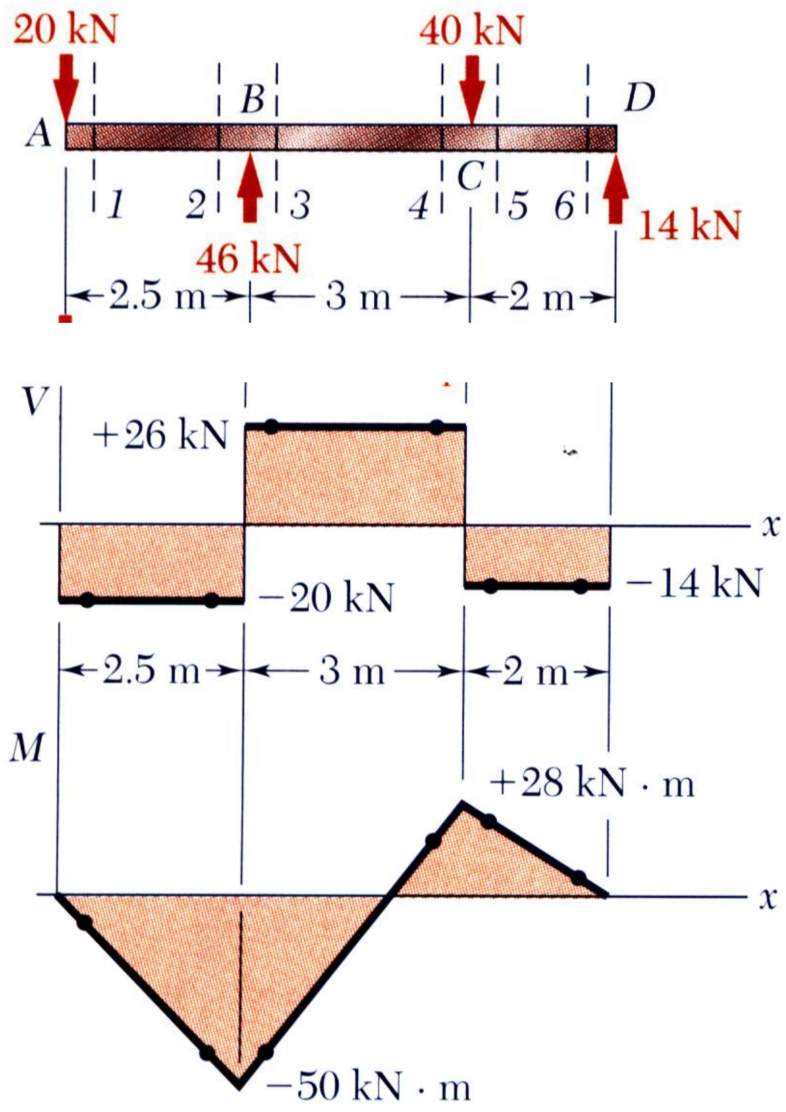
$$V_3 = +26\,\text{kN} \quad M_3 = -50\,\text{kN}\cdot\text{m}$$

$$V_4 = +26\,\text{kN} \quad M_4 = +28\,\text{kN}\cdot\text{m}$$

$$V_5 = -14\,\text{kN} \quad M_5 = +28\,\text{kN}\cdot\text{m}$$

$$V_6 = -14\,\text{kN} \quad M_6 = 0$$

Sample Problem 5.1



- Identify the maximum shear and bending-moment from plots of their distributions.

$$V_m = 26\,\text{kN} \quad M_m = |M_B| = 50\,\text{kN}\cdot\text{m}$$

- Apply the elastic flexure formulas to determine the corresponding maximum normal stress.

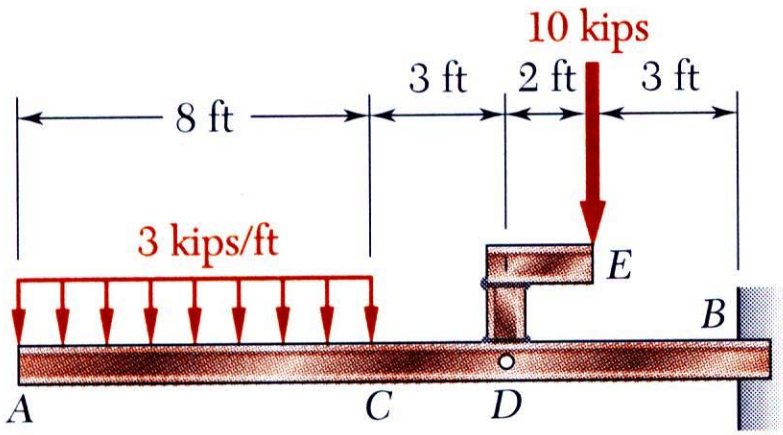
$$S = \tfrac{1}{6}bh^2 = \tfrac{1}{6}(0.080\,\text{m})(0.250\,\text{m})^2$$

$$= 833.33\times10^{-6}\,\text{m}^3$$

$$\sigma_m = \frac{|M_B|}{S} = \frac{50\times10^3\,\text{N}\cdot\text{m}}{833.33\times10^{-6}\,\text{m}^3}$$

$$\boxed{\sigma_m = 60.0\times10^6\,\text{Pa}}$$

Sample Problem 5.2



The structure shown is constructed of a W10x112 rolled-steel beam. (*a*) Draw the shear and bending-moment diagrams for the beam and the given loading. (*b*) determine normal stress in sections just to the right and left of point *D*.

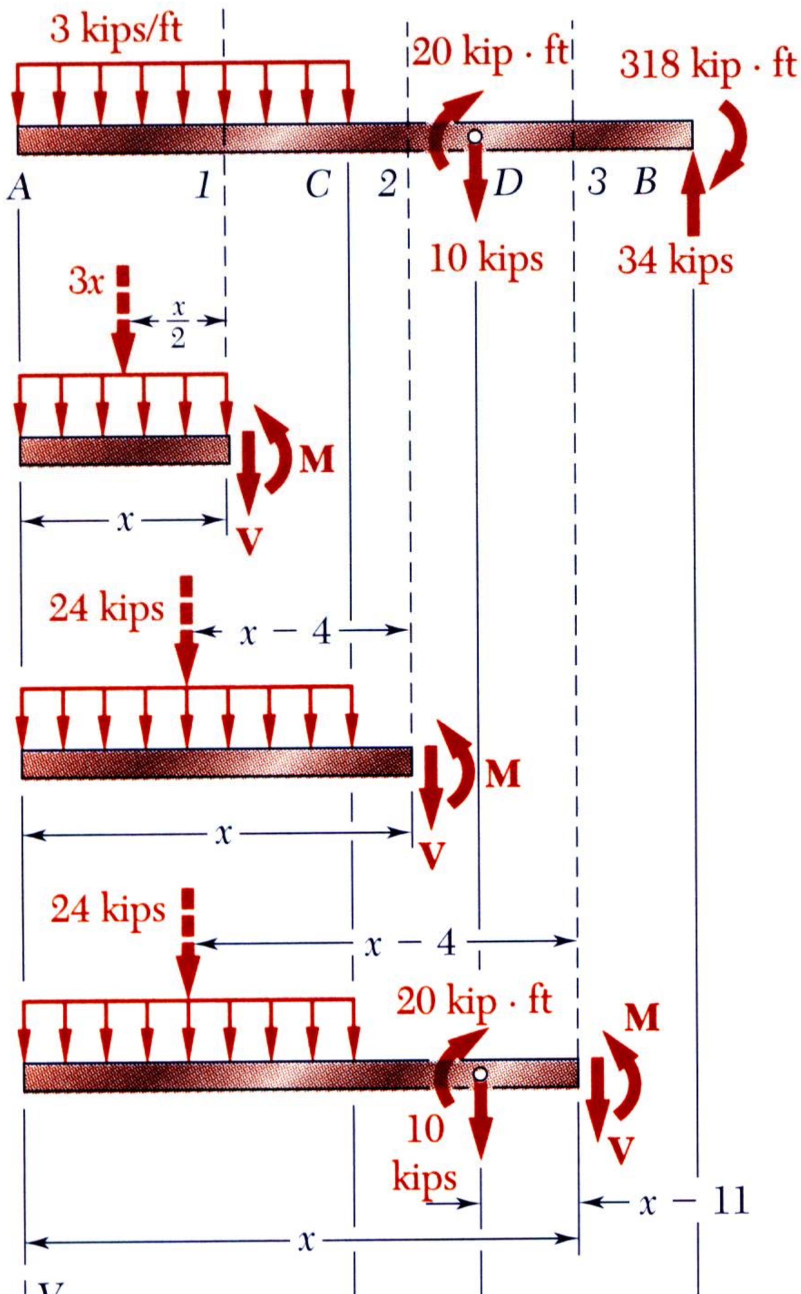
SOLUTION:

- Replace the 10 kip load with an equivalent force-couple system at *D*. Find the reactions at *B* by considering the beam as a rigid body.

- Section the beam at points near the support and load application points. Apply equilibrium analyses on resulting free-bodies to determine internal shear forces and bending couples.

- Apply the elastic flexure formulas to determine the maximum normal stress to the left and right of point *D*.

Sample Problem 5.2



SOLUTION:

- Replace the 10 kip load with equivalent force-couple system at *D*. Find reactions at *B*.
- Section the beam and apply equilibrium analyses on resulting free-bodies.

From A to C:

$$\sum F_y = 0 \quad -3x - V = 0 \qquad V = -3x \text{ kips}$$

$$\sum M_1 = 0 \quad (3x)\left(\tfrac{1}{2}x\right) + M = 0 \quad M = -1.5x^2 \text{ kip} \cdot \text{ft}$$

From C to D:

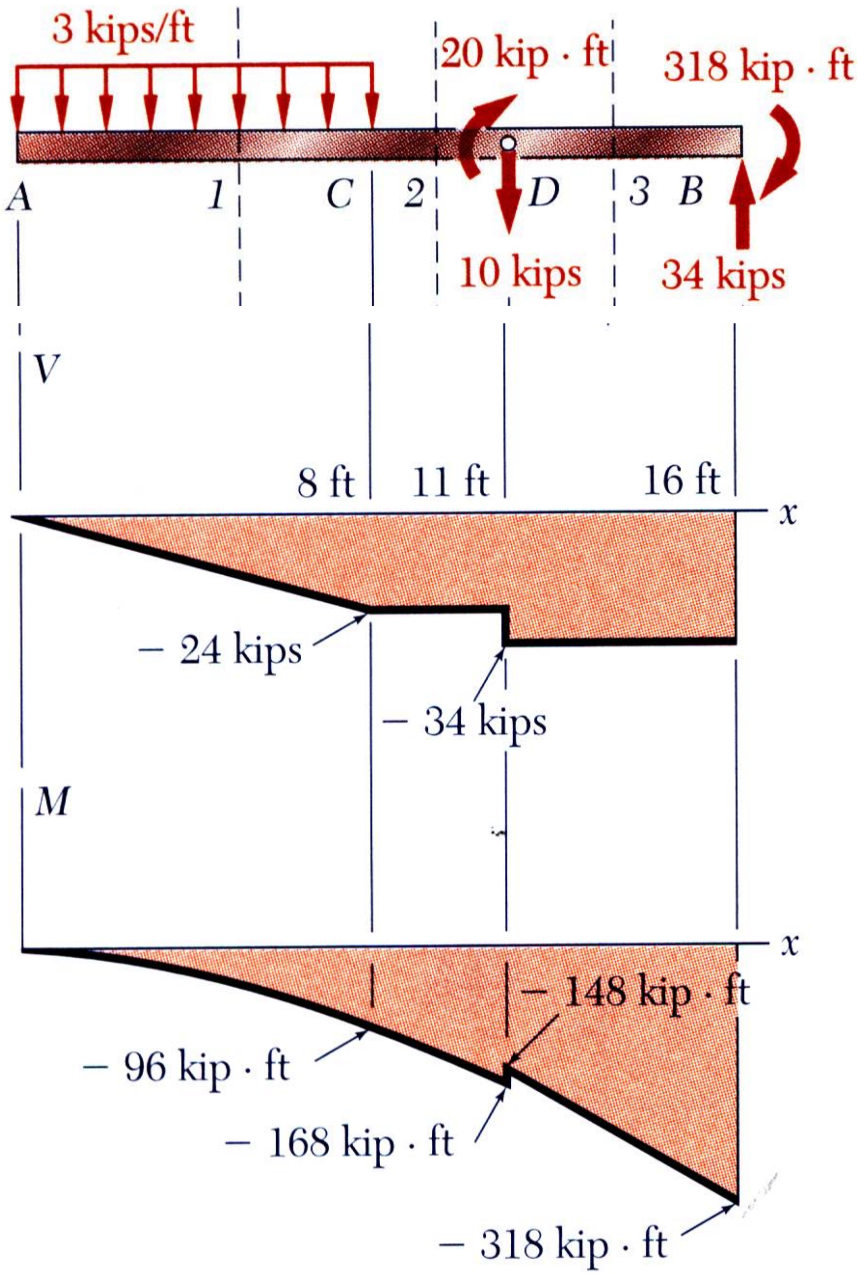
$$\sum F_y = 0 \quad -24 - V = 0 \qquad V = -24 \text{ kips}$$

$$\sum M_2 = 0 \quad 24(x-4) + M = 0 \quad M = (96 - 24x) \text{ kip} \cdot \text{ft}$$

From D to B:

$$V = -34 \text{ kips} \qquad M = (226 - 34x) \text{ kip} \cdot \text{ft}$$

Sample Problem 5.2



- Apply the elastic flexure formulas to determine the maximum normal stress to the left and right of point *D*.

From Appendix C for a W10x112 rolled steel shape, $S = 126\ \text{in}^3$ about the *X-X* axis.

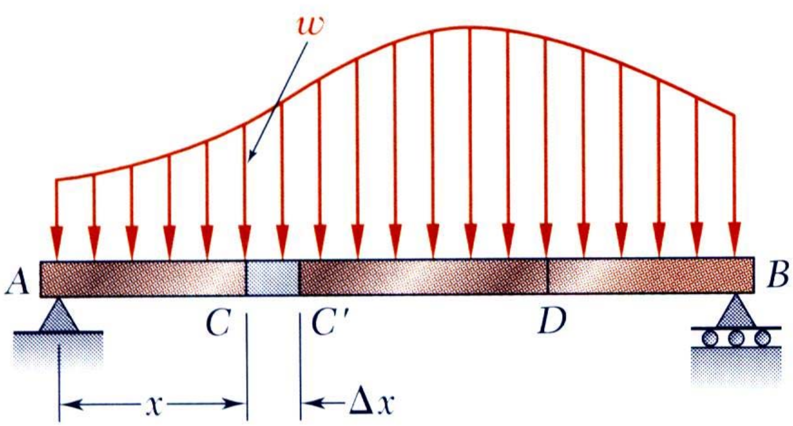
To the left of D:

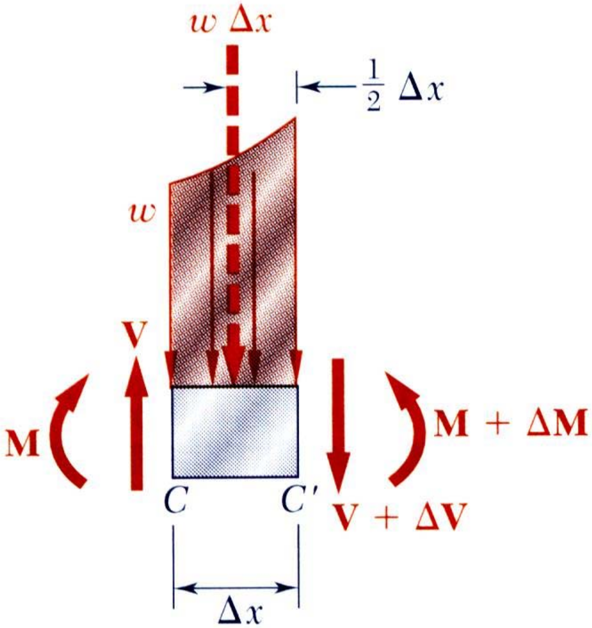
$$\sigma_m = \frac{|M|}{S} = \frac{2016\,\text{kip}\cdot\text{in}}{126\,\text{in}^3} \qquad \boxed{\sigma_m = 16.0\,\text{ksi}}$$

To the right of D:

$$\sigma_m = \frac{|M|}{S} = \frac{1776\,\text{kip}\cdot\text{in}}{126\,\text{in}^3} \qquad \boxed{\sigma_m = 14.1\,\text{ksi}}$$

Relations Among Load, Shear, and Bending Moment





- Relationship between load and shear:

$$\sum F_y = 0: \quad V - (V + \Delta V) - w\Delta x = 0$$

$$\Delta V = -w\,\Delta x$$

$$\frac{dV}{dx} = -w$$

$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx$$

- Relationship between shear and bending moment:

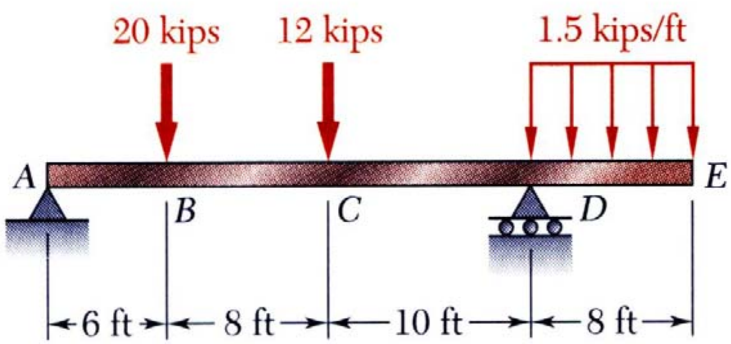
$$\sum M_{C'} = 0: \quad (M + \Delta M) - M - V\,\Delta x + w\Delta x \frac{\Delta x}{2} = 0$$

$$\Delta M = V\,\Delta x - \tfrac{1}{2} w (\Delta x)^2$$

$$\frac{dM}{dx} = 0$$

$$M_D - M_C = \int_{x_C}^{x_D} V\,dx$$

Sample Problem 5.3

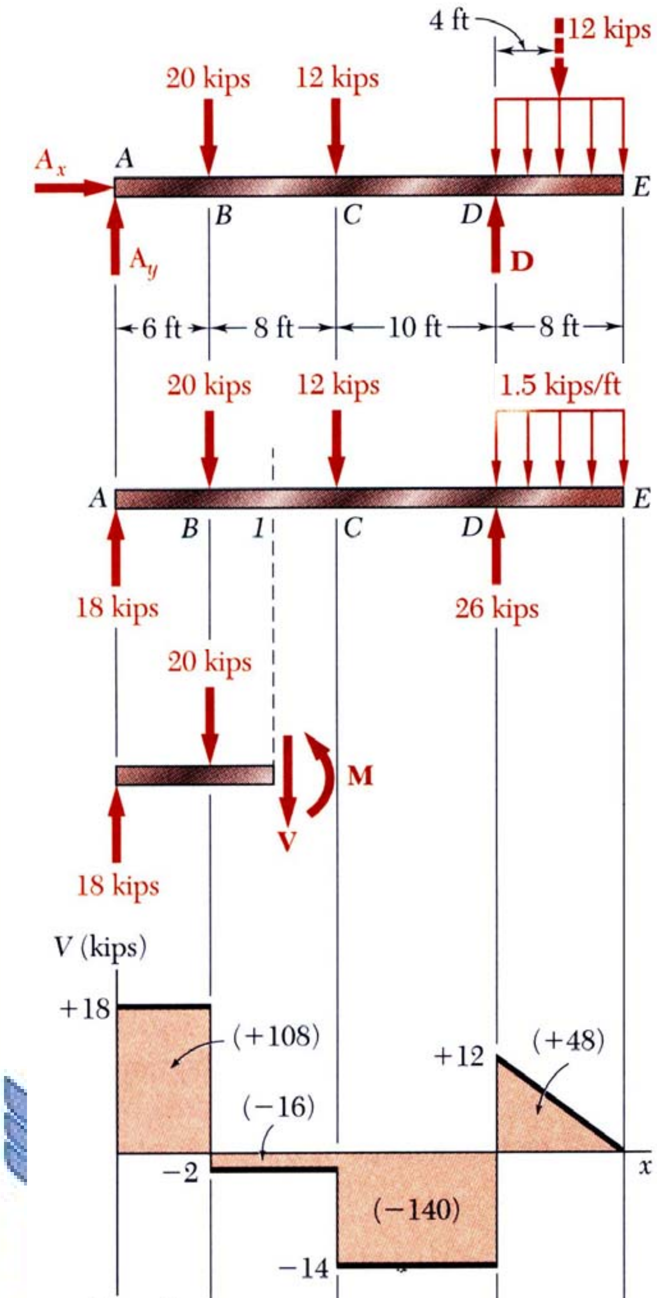


Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *A* and *D*.
- Apply the relationship between shear and load to develop the shear diagram.
- Apply the relationship between bending moment and shear to develop the bending moment diagram.

Sample Problem 5.3



SOLUTION:

- Taking the entire beam as a free body, determine the reactions at A and D.

$$\sum M_A = 0$$

$$0 = D(24\,\text{ft}) - (20\,\text{kips})(6\,\text{ft}) - (12\,\text{kips})(14\,\text{ft}) - (12\,\text{kips})(28\,\text{ft})$$

$$D = 26\,\text{kips}$$

$$\sum F_y = 0$$

$$0 = A_y - 20\,\text{kips} - 12\,\text{kips} + 26\,\text{kips} - 12\,\text{kips}$$

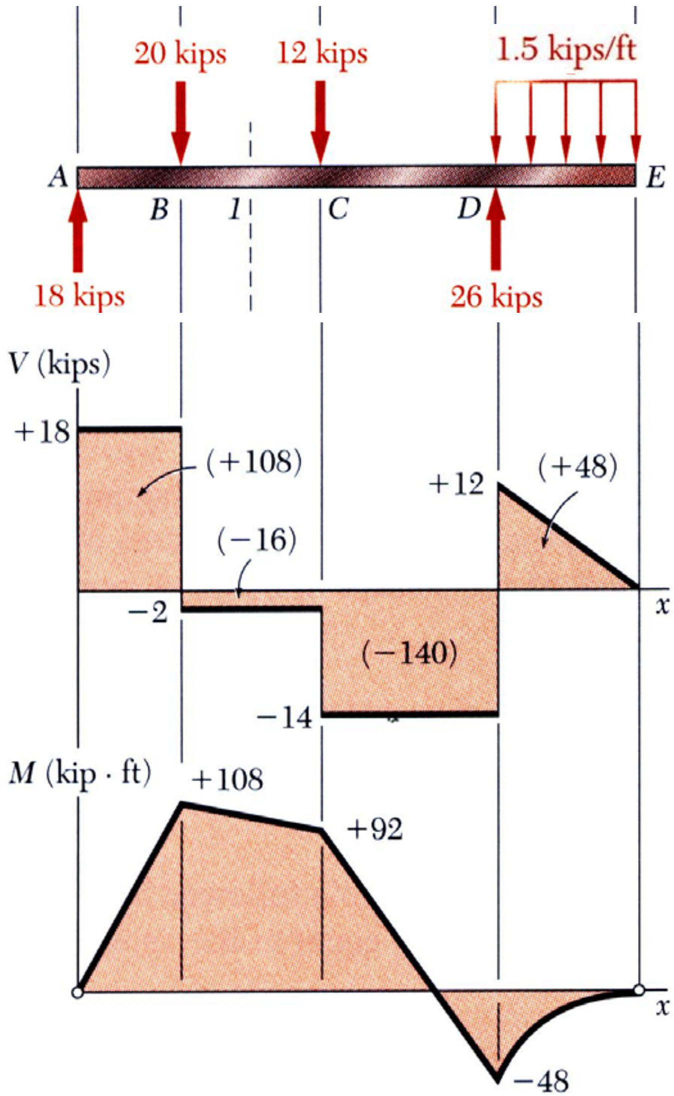
$$A_y = 18\,\text{kips}$$

- Apply the relationship between shear and load to develop the shear diagram.

$$\frac{dV}{dx} = -w \qquad dV = -w\,dx$$

- zero slope between concentrated loads
- linear variation over uniform load segment

Sample Problem 5.3

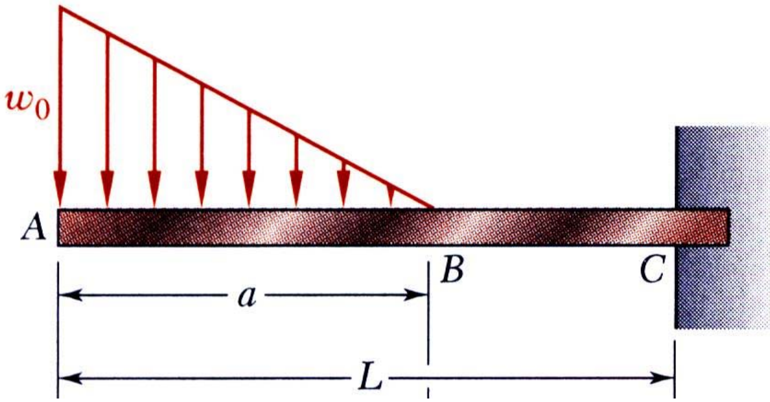


- Apply the relationship between bending moment and shear to develop the bending moment diagram.

$$\frac{dM}{dx} = V \qquad dM = V\,dx$$

- bending moment at *A* and *E* is zero
- bending moment variation between *A, B, C* and *D* is linear
- bending moment variation between D and *E* is quadratic
- net change in bending moment is equal to areas under shear distribution segments
- total of all bending moment changes across the beam should be zero

Sample Problem 5.5

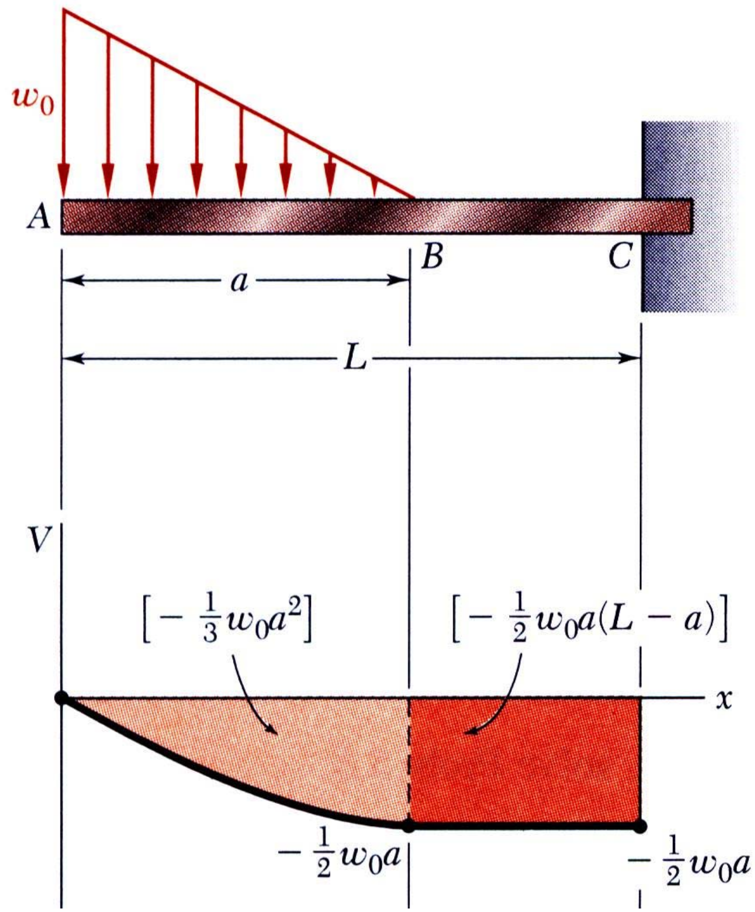


Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *C*.
- Apply the relationship between shear and load to develop the shear diagram.
- Apply the relationship between bending moment and shear to develop the bending moment diagram.

Sample Problem 5.5



SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *C*.

$$\sum F_y = 0 = -\tfrac{1}{2}w_0a + R_C \qquad R_C = \tfrac{1}{2}w_0a$$

$$\sum M_C = 0 = \tfrac{1}{2}w_0a\left(L-\frac{a}{3}\right) + M_C \qquad M_C = -\tfrac{1}{2}w_0a\left(L-\frac{a}{3}\right)$$

Results from integration of the load and shear distributions should be equivalent.

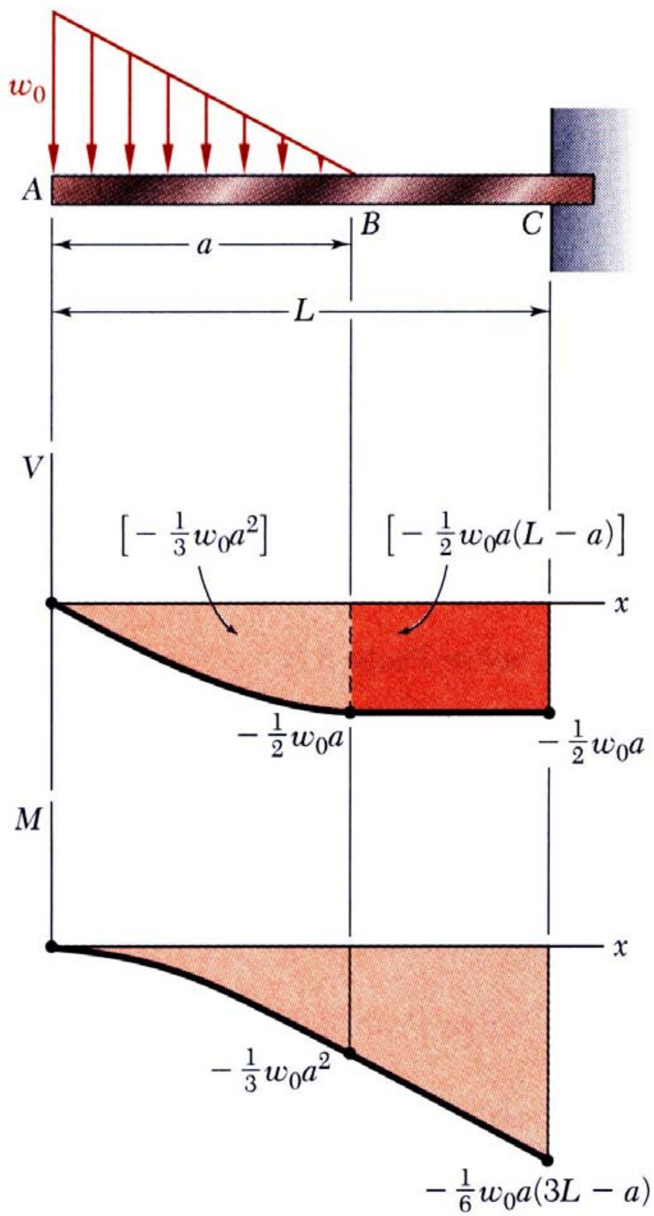
- Apply the relationship between shear and load to develop the shear diagram.

$$V_B - V_A = -\int_0^a w_0\left(1-\frac{x}{a}\right)dx = -\left[w_0\left(x-\frac{x^2}{2a}\right)\right]_0^a$$

$$V_B = -\tfrac{1}{2}w_0a = -(\textit{area under load curve})$$

- No change in shear between *B* and *C*.
- Compatible with free body analysis

Sample Problem 5.5



- Apply the relationship between bending moment and shear to develop the bending moment diagram.

$$M_B - M_A = \int_0^a \left(-w_0\left(x - \frac{x^2}{2a}\right)\right)dx = \left[-w_0\left(\frac{x^2}{2} - \frac{x^3}{6a}\right)\right]_0^a$$

$$M_B = -\tfrac{1}{3}w_0a^2$$

$$M_B - M_C = \int_a^L \left(-\tfrac{1}{2}w_0a\right)dx = -\tfrac{1}{2}w_0a(L-a)$$

$$M_C = -\tfrac{1}{6}w_0a(3L-a) = \frac{a\,w_0}{2}\left(L - \frac{a}{3}\right)$$

Results at *C* are compatible with free-body analysis

Design of Prismatic Beams for Bending

- The largest normal stress is found at the surface where the maximum bending moment occurs.

$$\sigma_m = \frac{|M|_{\max} c}{I} = \frac{|M|_{\max}}{S}$$

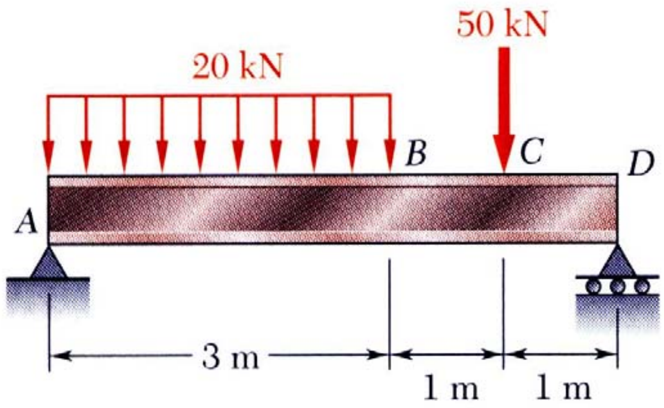
- A safe design requires that the maximum normal stress be less than the allowable stress for the material used. This criteria leads to the determination of the minimum acceptable section modulus.

$$\sigma_m \leq \sigma_{all}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{all}}$$

- Among beam section choices which have an acceptable section modulus, the one with the smallest weight per unit length or cross sectional area will be the least expensive and the best choice.

Sample Problem 5.8

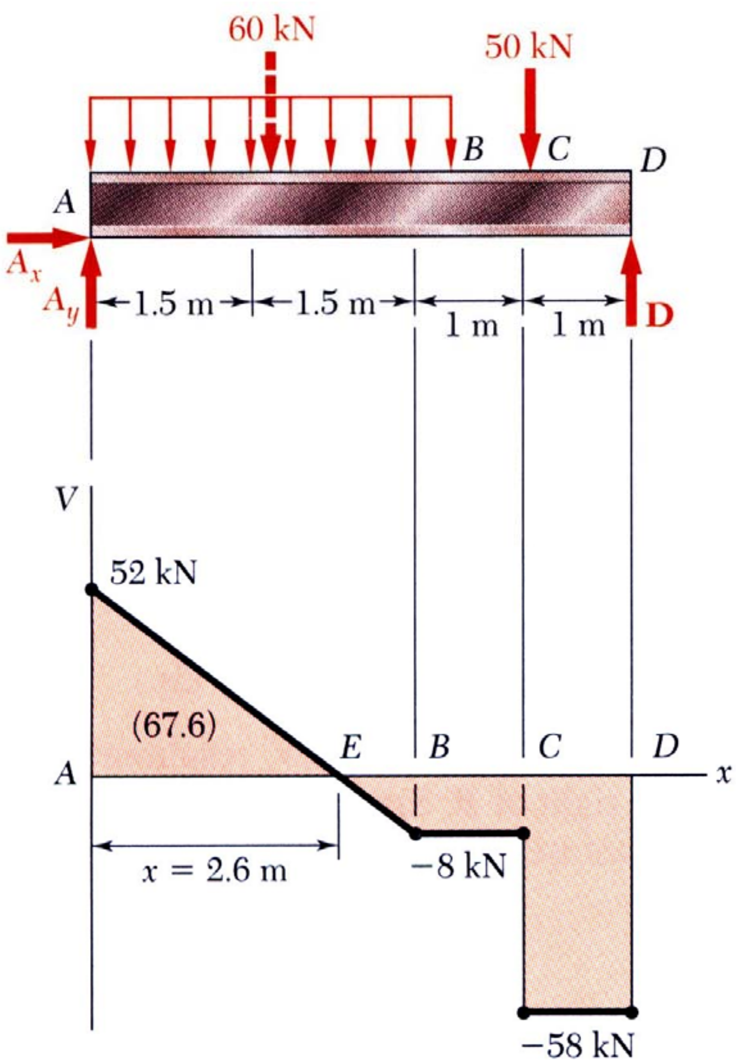


A simply supported steel beam is to carry the distributed and concentrated loads shown. Knowing that the allowable normal stress for the grade of steel to be used is 160 MPa, select the wide-flange shape that should be used.

SOLUTION:

- Considering the entire beam as a free-body, determine the reactions at *A* and *D*.
- Develop the shear diagram for the beam and load distribution. From the diagram, determine the maximum bending moment.
- Determine the minimum acceptable beam section modulus. Choose the best standard section which meets this criteria.

Sample Problem 5.8



- Considering the entire beam as a free-body, determine the reactions at *A* and *D*.

$$\sum M_A = 0 = D(5\,\text{m}) - (60\,\text{kN})(1.5\,\text{m}) - (50\,\text{kN})(4\,\text{m})$$

$$D = 58.0\,\text{kN}$$

$$\sum F_y = 0 = A_y + 58.0\,\text{kN} - 60\,\text{kN} - 50\,\text{kN}$$

$$A_y = 52.0\,\text{kN}$$

- Develop the shear diagram and determine the maximum bending moment.

$$V_A = A_y = 52.0\,\text{kN}$$

$$V_B - V_A = -(\text{area under load curve}) = -60\,\text{kN}$$

$$V_B = -8\,\text{kN}$$

- Maximum bending moment occurs at $V = 0$ or $x = 2.6$ m.

$$|M|_{max} = (\text{area under shear curve, } A \text{ to } E)$$

$$= 67.6\,\text{kN}$$

Sample Problem 5.8

- Determine the minimum acceptable beam section modulus.

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{all}} = \frac{67.6\,\text{kN}\cdot\text{m}}{160\,\text{MPa}}$$

$$= 422.5\times10^{-6}\,\text{m}^3 = 422.5\times10^3\,\text{mm}^3$$

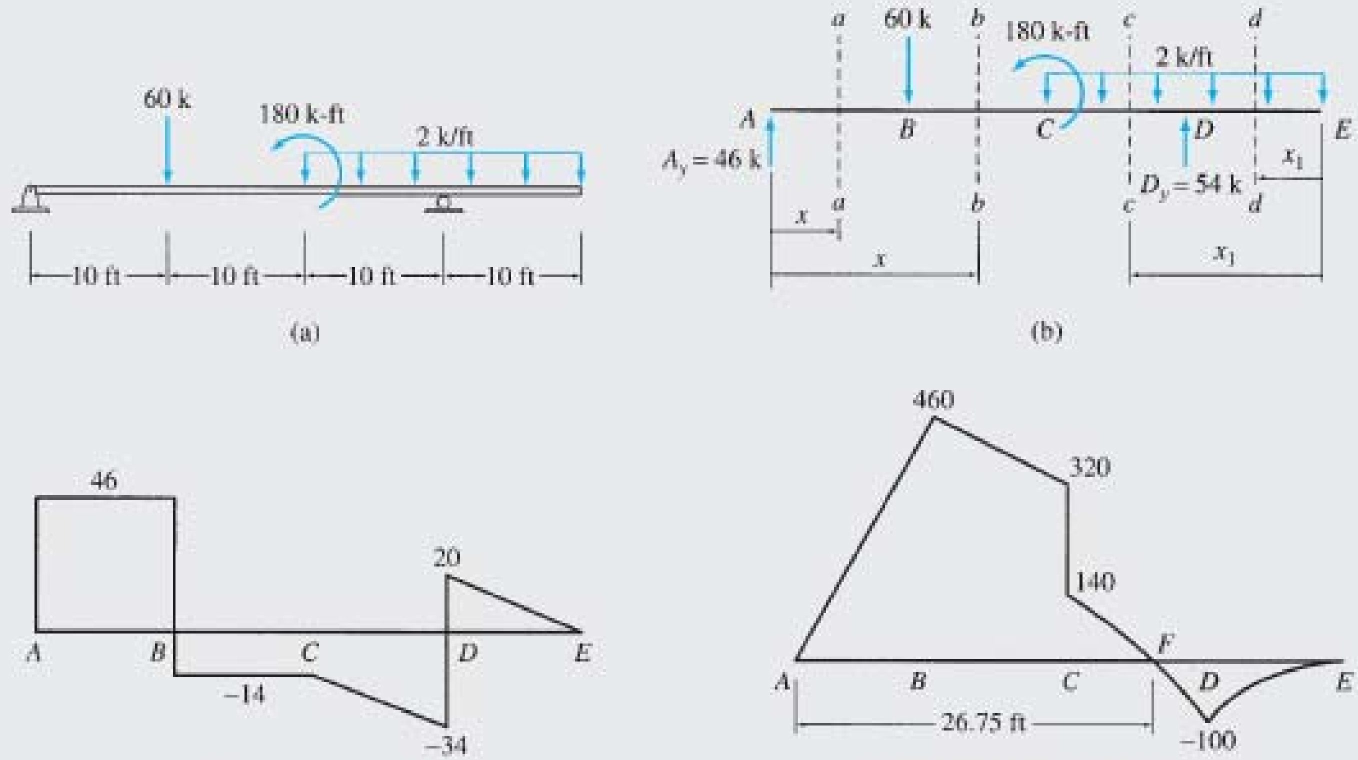
- Choose the best standard section which meets this criteria.

$W360\times32.9$

Shape	S, mm^3
W410×38.8	637
W360×32.9	474
W310×38.7	549
W250×44.8	535
W200×46.1	448

Example 5.3

Draw the shear and bending moment diagrams for the beam shown in Fig. 5.5(a).



(c) Shear Diagram (k)

(d) Bending Moment Diagram (k-ft)

FIG. 5.5

Solution

Reactions. See Fig. 5.5(b).

$$+ \rightarrow \sum F_x = 0 \qquad A_x = 0$$

$$+ \circlearrowleft \sum M_D = 0$$

$$-A_y(30) + 60(20) + 180 + 2(20)(0) = 0$$

$$A_y = 46 \text{ k} \uparrow$$

continued

$$+\uparrow \sum F_y = 0$$

$$46 - 60 - 2(20) + D_y = 0$$

$$D_y = 54 \text{ k} \uparrow$$

Shear Diagram. To determine the equation for shear in segment AB of the beam, we pass a section aa at a distance x from support A, as shown in Fig. 5.5(b). Considering the free body to the left of this section, we obtain

$$S = 46 \text{ k} \quad \text{for } 0 < x < 10 \text{ ft}$$

As this equation indicates, the shear is constant at 46 k from an infinitesimal distance to the right of point A to an infinitesimal distance to the left of point B. At point A, the shear increases abruptly from 0 to 46 k, so a vertical line is drawn from 0 to 46 on the shear diagram (Fig. 5.5(c)) at A to indicate this change. This is followed by a horizontal line from A to B to indicate that the shear remains constant in this segment.

Next, by using section bb (Fig. 5.5(b)), we determine the equation for shear in segment BC as

$$S = 46 - 60 = -14 \text{ k} \quad \text{for } 10 \text{ ft} < x \leq 20 \text{ ft}$$

The abrupt change in shear from 46 k at an infinitesimal distance to the left of B to -14 k at an infinitesimal distance to the right of B is shown on the shear diagram (Fig. 5.5(c)) by a vertical line from +46 to -14. A horizontal line at -14 is then drawn from B to C to indicate that the shear remains constant at this value throughout this segment.

To determine the equations for shear in the right half of the beam, it is convenient to use another coordinate, x_1, directed to the left from the end E of the beam, as shown in Fig. 5.5(b). The equations for shear in segments ED and DC are obtained by considering the free bodies to the right of sections dd and cc, respectively. Thus,

$$S = 2x_1 \quad \text{for } 0 \leq x_1 < 10 \text{ ft}$$

and

$$S = 2x_1 - 54 \quad \text{for } 10 \text{ ft} < x_1 \leq 20 \text{ ft}$$

These equations indicate that the shear increases linearly from zero at E to +20 k at an infinitesimal distance to the right of D; it then drops abruptly to -34 k at an infinitesimal distance to the left of D; and from there it increases linearly to -14 k at C. This information is plotted on the shear diagram, as shown in Fig. 5.5(c). **Ans.**

Bending Moment Diagram. Using the same sections and coordinates employed previously for computing shear, we determine the following equations for bending moment in the four segments of the beam. For segment AB:

$$M = 46x \quad \text{for } 0 \leq x \leq 10 \text{ ft}$$

For segment BC:

$$M = 46x - 60(x - 10) = -14x + 600 \quad \text{for } 10 \text{ ft} \leq x < 20 \text{ ft}$$

For segment ED:

$$M = -2x_1\left(\frac{x_1}{2}\right) = -x_1^2 \quad \text{for } 0 \leq x_1 \leq 10 \text{ ft}$$

For segment DC:

$$M = -x_1^2 + 54(x_1 - 10) = -x_1^2 + 54x_1 - 540 \quad \text{for } 10 \text{ ft} \leq x_1 < 20 \text{ ft}$$

The first two equations, for the left half of the beam, indicate that the bending moment increases linearly from 0 at A to 460 k-ft at B; it then decreases linearly to 320 k-ft at C, as shown on the bending moment diagram in Fig. 5.5(d). The last two equations for the right half of the beam are quadratic in x_1. The values of M computed from these equations are plotted on the bending moment diagram shown in Fig. 5.5(d). It can be seen that M decreases from 0 at E to -100 k-ft at D, and it then increases to +140 k-ft at an infinitesimal distance to the right of C. Note that at C, the bending moment drops abruptly by an amount $320 - 140 = 180$ k-ft, which is equal to the magnitude of the moment of the counterclockwise external couple acting at this point.

continued

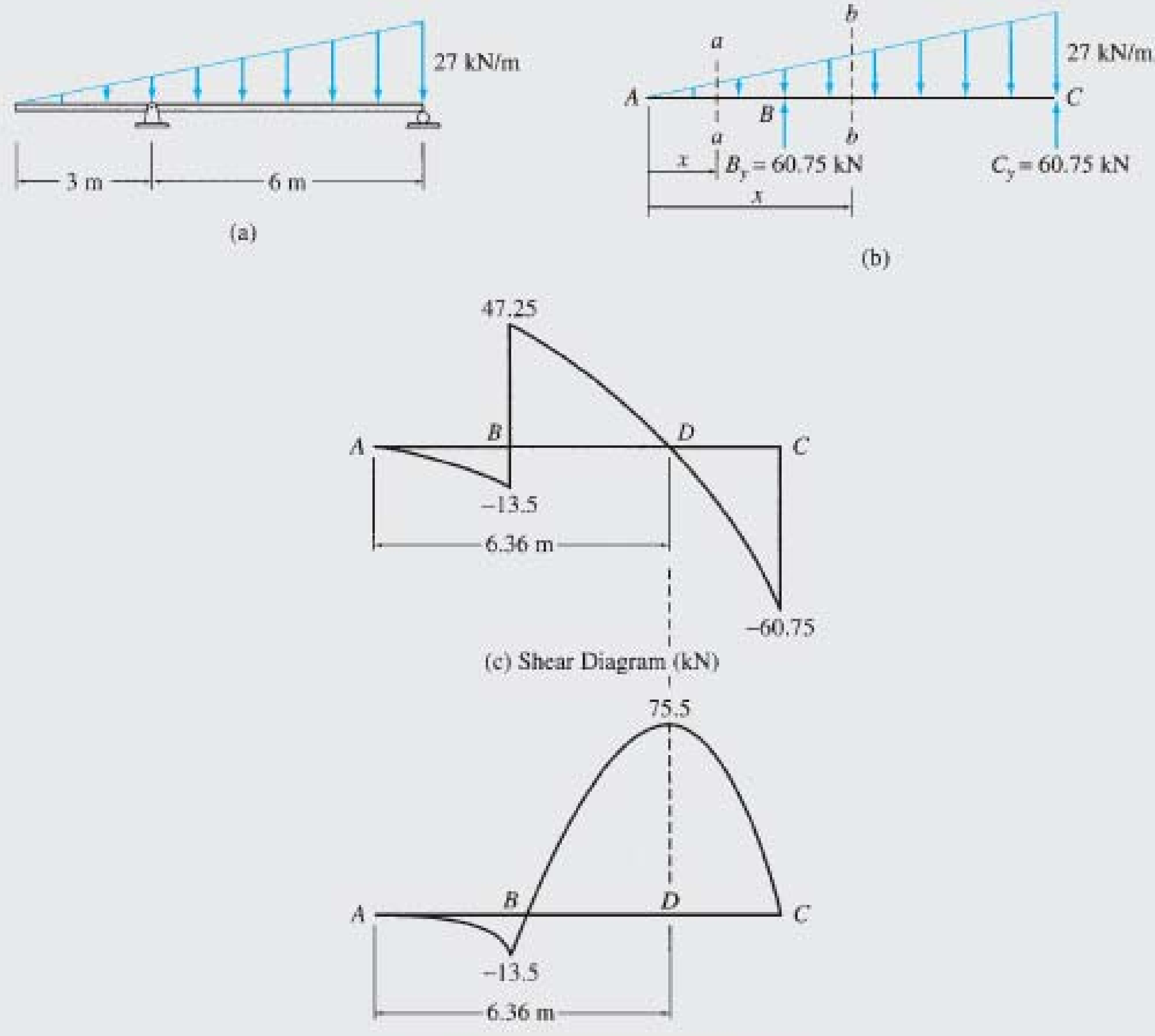
A point at which the bending moment is zero is termed the *point of inflection*. To determine the location of the point of inflection F (Fig. 5.5(d)), we set $M = 0$ in the equation for bending moment in segment DC to obtain

$$M = -x_1^2 + 54x_1 - 540 = 0$$

from which $x_1 = 13.25$ ft; that is, point F is located at a distance of 13.25 ft from end E, or $40 - 13.25 = 26.75$ ft from support A of the beam, as shown in Fig. 5.5(d). **Ans.**

Example 5.4

Draw the shear and bending moment diagrams for the beam shown in Fig. 5.6(a).



(a) (b)

(c) Shear Diagram (kN)

(d) Bending Moment Diagram (kN · m)

FIG. 5.6

continued

Solution

Reactions. See Fig. 5.6(b).

$$+\rightarrow \sum F_x = 0 \qquad B_x = 0$$

$$+\circlearrowleft \sum M_c = 0$$

$$\left(\frac{1}{2}\right)(9)(27)\left(\frac{9}{3}\right) - B_y(6) = 0 \qquad B_y = 60.75 \text{ kN} \uparrow$$

$$+\uparrow \sum F_y = 0$$

$$-\left(\frac{1}{2}\right)(9)(27) + 60.75 + C_y = 0 \qquad C_y = 60.75 \text{ kN} \uparrow$$

Shear Diagram. To determine the equations for shear in segments AB and BC of the beam, we pass sections aa and bb through the beam, as shown in Fig. 5.6(b). Considering the free bodies to the left of these sections and realizing that the load intensity, $w(x)$, at a point at a distance x from end A is $w(x) = (\frac{27}{9})x = 3x$ kN/m, we obtain the following equations for shear in segments AB and BC, respectively:

$$S = -\left(\frac{1}{2}\right)(x)(3x) = -\frac{3x^2}{2} \quad \text{for } 0 \le x < 3 \text{ m}$$

$$S = -\left(\frac{3x^2}{2}\right) + 60.75 \quad \text{for } 3 \text{ m} < x < 9 \text{ m}$$

The values of S computed from these equations are plotted to obtain the shear diagram shown in Fig. 5.6(c). The point D at which the shear is zero is obtained from the equation

$$S = -\left(\frac{3x^2}{2}\right) + 60.75 = 0$$

from which $x = 6.36$ m. **Ans.**

Bending Moment Diagram. Using the same sections employed previously for computing shear, we determine the following equations for bending moment in segments AB and BC, respectively:

$$M = -\left(\frac{1}{2}\right)(x)(3x)\left(\frac{x}{3}\right) = -\frac{x^3}{2} \quad \text{for } 0 \le x \le 3 \text{ m}$$

$$M = -\left(\frac{x^3}{2}\right) + 60.75(x-3) \quad \text{for } 3 \text{ m} \le x \le 9 \text{ m}$$

The values of M computed from these equations are plotted to obtain the bending moment diagram shown in Fig. 5.6(d). To locate the point at which the bending moment is maximum, we differentiate the equation for M in segment BC with respect to x and set the derivative dM/dx equal to zero; that is,

$$\frac{dM}{dx} = \left(-\frac{3x^2}{2}\right) + 60.75 = 0$$

from which $x = 6.36$ m. This indicates that the maximum bending moment occurs at the same point at which the shear is zero. Also, a comparison of the expressions for dM/dx and S in segment BC indicates that the two equations are identical; that is, the slope of the bending moment diagram at a point is equal to the shear at that point. (This relationship, which is generally valid, is discussed in detail in a subsequent section.)

Finally, the magnitude of the maximum moment is determined by substituting $x = 6.36$ m into the equation for M in segment BC:

$$M_{max} = -\left[\frac{(6.36)^3}{2}\right] + 60.75(6.36 - 3) = 75.5 \text{ kN} \cdot \text{m}$$ **Ans.**

Example 5.5

Draw the shear and bending moment diagrams and the qualitative deflected shape for the beam shown in Fig. 5.9(a).

Solution

Reactions. (See Fig. 5.9(b).)

$$+\rightarrow \sum F_x = 0 \qquad A_x = 0$$

continued

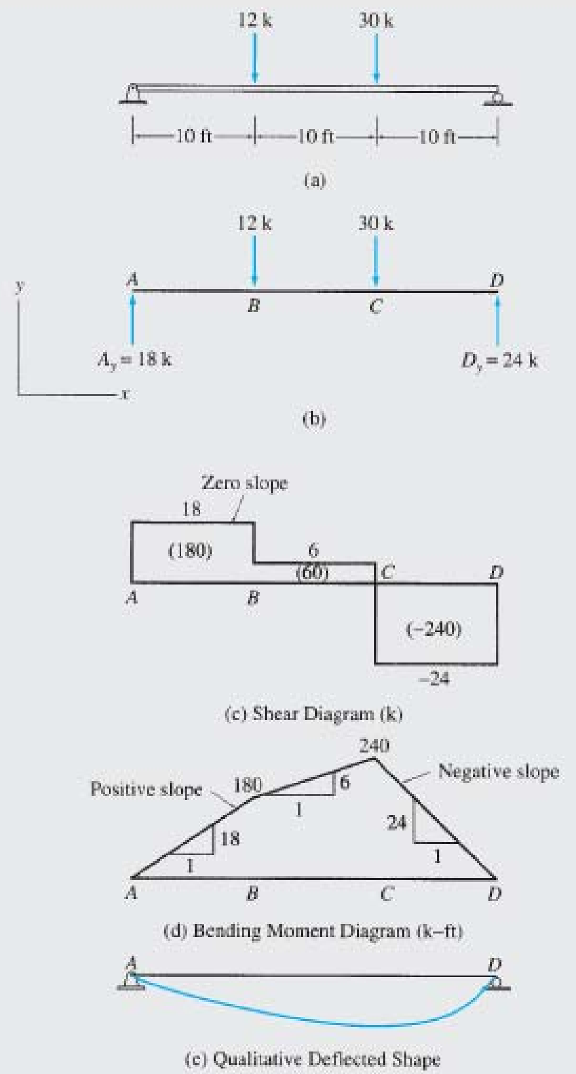


FIG. 5.9

By proportions,

$$A_y = 12\left(\frac{20}{30}\right) + 30\left(\frac{10}{30}\right) = 18 \text{ k} \qquad A_y = 18 \text{ k} \uparrow$$

$$+\uparrow \sum F_y = 0$$

$$18 - 12 - 30 + D_y = 0$$

$$D_y = 24 \text{ k} \qquad D_y = 24 \text{ k} \uparrow$$

continued

Shear Diagram.

Point *A*. Since a positive (upward) concentrated force of 18-k magnitude acts at point A, the shear diagram increases abruptly from 0 to +18 k at this point.

Point *B*. The shear just to the left of point B is given by

$$S_{B,L} = S_{A,R} + \text{area under the load diagram between just to the right of } A \text{ to just to the left of } B$$

in which the subscripts ", L" and ", R" are used to denote "just to the left" and "just to the right," respectively. As no load is applied to this segment of the beam,

$$S_{B,L} = 18 + 0 = 18 \text{ k}$$

Because a negative (downward) concentrated load of 12-k magnitude acts at point B, the shear just to the right of B is

$$S_{B,R} = 18 - 12 = 6 \text{ k}$$

Point *C*.

$$S_{C,L} = S_{B,R} + \text{area under the load diagram between just to the right of } B \text{ to just to the left of } C$$

$$S_{C,L} = 6 + 0 = 6 \text{ k}$$

$$S_{C,R} = 6 - 30 = -24 \text{ k}$$

Point *D*.

$$S_{D,L} = -24 + 0 = -24 \text{ k}$$

$$S_{D,R} = -24 + 24 = 0$$ Checks

The numerical values of shear computed at points A, B, C, and D are used to construct the shear diagram as shown in Fig. 5.9(c). The shape of the diagram between these ordinates has been established by applying Eq. (5.3), which states that the slope of the shear diagram at a point is equal to the load intensity at that point. Because no load is applied to the beam between these points, the slope of the shear diagram is zero between these points, and the shear diagram consists of a series of horizontal lines, as shown in the figure. Note that the shear diagram closes (i.e., returns to zero) just to the right of the right end D of the beam, indicating that the analysis has been carried out correctly. Ans.

To facilitate the construction of the bending moment diagram, the areas of the various segments of the shear diagram have been computed and are shown in parentheses on the shear diagram (Fig. 5.9(c)).

Bending Moment Diagram.

Point *A*. Because no couple is applied at end A, $M_A = 0$.

Point *B*.

$$M_B = M_A + \text{area under the shear diagram between } A \text{ and } B$$

$$M_B = 0 + 180 = 180 \text{ k-ft}$$

Point *C*. $$M_C = 180 + 60 = 240 \text{ k-ft}$$

Point *D*. $$M_D = 240 - 240 = 0$$ Checks

The numerical values of bending moment computed at points A, B, C, and D are used to construct the bending moment diagram shown in Fig. 5.9(d). The shape of the diagram between these ordinates has been established by applying Eq. (5.8), which states that the slope of the bending moment diagram at a point is equal to the shear at that point. As the shear between these points is constant, the slope of the bending moment diagram must be constant between these points.

continued

Therefore, the ordinates of the bending moment diagram are connected by straight, sloping lines. In segment *AB*, the shear is +18 k. Therefore, the slope of the bending moment diagram in this segment is 18:1, and it is positive—that is, *upward to the right* (/). In segment *BC*, the shear drops to +6 k; therefore, the slope of the bending moment diagram reduces to 6:1 but remains positive. In segment *CD*, the shear becomes −24; consequently, the slope of the bending moment diagram becomes negative—that is, *downward to the right* (\), as shown in Fig. 5.9(d). Note that the maximum bending moment occurs at point *C*, where the shear changes from positive to the left to negative to the right. **Ans.**

Qualitative Deflected Shape. A qualitative deflected shape of the beam is shown in Fig. 5.9(e). As the bending moment is positive over its entire length, the beam bends concave upward, as shown. **Ans.**

Example 5.6

Draw the shear and bending moment diagrams and the qualitative deflected shape for the beam shown in Fig. 5.10(a).

Solution

Reactions. (See Fig. 5.10(b).)

$$+\rightarrow \sum F_x = 0 \qquad A_x = 0$$

$$+\uparrow \sum F_y = 0$$

$$A_y - 70 = 0$$

$$A_y = 70 \text{ kN} \qquad A_y = 70 \text{ kN} \uparrow$$

$$+\circlearrowleft \sum M_A = 0$$

$$M_A - 70(6) - 200 = 0$$

$$M_A = 620 \text{ kN}\cdot\text{m} \qquad M_A = 620 \text{ kN}\cdot\text{m} \circlearrowleft$$

Shear Diagram.

Point *A*. $S_{A,R} = 70$ kN

Point *B*. $S_{B,L} = 70 + 0 = 70$ kN

$S_{B,R} = 70 - 70 = 0$

Point *C*. $S_{C,L} = 0 + 0 = 0$

$S_{C,R} = 0 + 0 = 0$ Checks

The numerical values of shear evaluated at points A, B, and C are used to construct the shear diagram as shown in Fig. 5.10(c). Because no load is applied to the beam between these points, the slope of the shear diagram is zero between these points. To facilitate the construction of the bending moment diagram, the area of the segment AB of the shear diagram has been computed and is shown in parentheses on the shear diagram (Fig. 5.10(c)). Ans.

continued

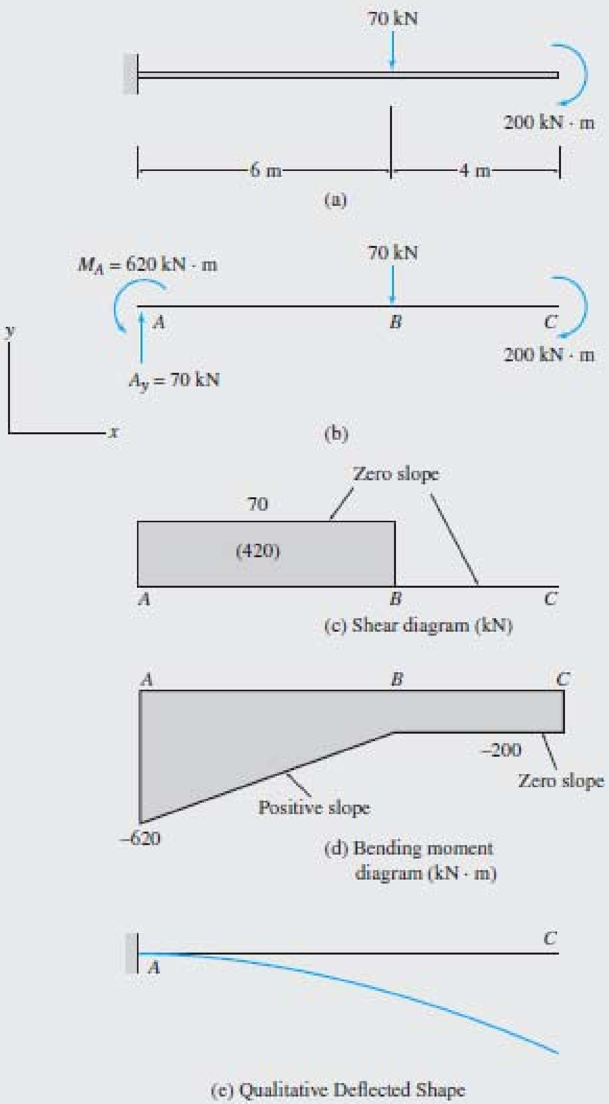


FIG. 5.10

Bending Moment Diagram.

Point A. Since a negative (counterclockwise) couple of 620 kN · m moment acts at point A, the bending moment diagram decreases abruptly from 0 to −620 kN · m at this point; that is,

$$M_{A,R} = -620 \text{ kN} \cdot \text{m}$$

Point B. $$M_B = -620 + 420 = -200 \text{ kN} \cdot \text{m}$$

continued

Point C.

$$M_{C,L} = -200 + 0 = -200 \text{ kN} \cdot \text{m}$$

$$M_{C,R} = -200 + 200 = 0 \qquad \text{Checks}$$

The bending moment diagram is shown in Fig. 5.10(d). The shape of this diagram between the ordinates just computed is based on the condition that the slope of the bending moment diagram at a point is equal to shear at that point. As the shear in the segments AB and BC is constant, the slope of the bending moment diagram must be constant in these segments. Therefore, the ordinates of the bending moment diagram are connected by straight lines. In segment AB, the shear is positive, and so is the slope of the bending moment diagram in this segment. In segment BC, the shear becomes zero; consequently, the slope of the bending moment diagram becomes zero, as shown in Fig. 5.10(d). **Ans.**

Qualitative Deflected Shape. A qualitative deflected shape of the beam is shown in Fig. 5.10(e). As the bending moment is negative over its entire length, the beam bends concave downward, as shown. **Ans.**

Example 5.7

Draw the shear and bending moment diagrams and the qualitative deflected shape for the beam shown in Fig. 5.11(a).

Solution

Reactions. (See Fig. 5.11(b).)

$$+\rightarrow \sum F_x = 0$$

$$A_x - 30 = 0$$

$$A_x = 30 \text{ kN} \qquad A_x = 30 \text{ kN} \rightarrow$$

$$+\circlearrowleft \sum M_D = 0$$

$$-A_y(27) + 10(15)(19.5) - 162 + 40(6) = 0$$

$$A_y = 111.22 \text{ kN} \qquad A_y = 111.22 \text{ kN} \uparrow$$

$$+\uparrow \sum F_y = 0$$

$$111.22 - 10(15) - 40 + D_y = 0$$

$$D_y = 78.78 \text{ kN} \qquad D_y = 78.78 \text{ kN} \uparrow$$

Shear Diagram.

Point *A*. $S_{A,R} = 111.22 \text{ kN}$

Point *B*. $S_B = 111.22 - 10(15) = -38.78 \text{ kN}$

Point *C*. $S_{C,L} = -38.78 + 0 = -38.78 \text{ kN}$

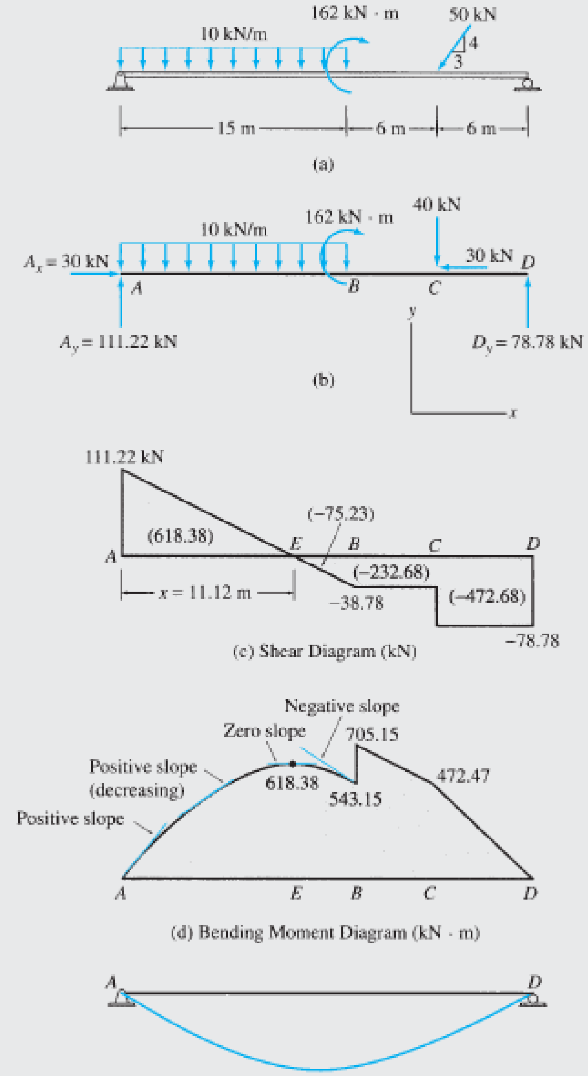
$S_{C,R} = -38.78 - 40 = -78.78 \text{ kN}$

Point *D*. $S_{D,L} = -78.78 + 0 = -78.78 \text{ kN}$

$S_{D,R} = -78.78 + 78.78 = 0$ Checks

The shear diagram is shown in Fig. 5.11(c). In segment AB, the beam is subjected to a downward (negative) uniformly distributed load of 10 kN/m. Because the load intensity is constant and negative in segment AB, the shear diagram in this segment is a straight line with negative slope. No distributed load is applied to the beam in segments BC and CD, so the shear diagram in these segments consists of horizontal lines, indicating zero slopes. **Ans.**

continued



(e) Qualitative Deflected Shape

FIG. 5.11

The point of zero shear, E, can be located by using the similar triangles forming the shear diagram between A and B. Thus,

$$\frac{x}{111.22} = \frac{15}{(111.22 + 38.78)}$$

$$x = 11.12 \text{ m}$$

continued

To facilitate the construction of the bending moment diagram, the areas of the various segments of the shear diagram have been computed; they are shown in parentheses on the shear diagram (Fig. 5.11(c)).

Bending Moment Diagram.

Point *A*. $M_A = 0$

Point *E*. $M_E = 0 + 618.38 = 618.38 \text{ kN} \cdot \text{m}$

Point *B*. $M_{B,L} = 618.38 - 75.23 = 543.15 \text{ kN} \cdot \text{m}$

$M_{B,R} = 543.15 + 162 = 705.15 \text{ kN} \cdot \text{m}$

Point *C*. $M_C = 705.15 - 232.68 = 472.47 \text{ kN} \cdot \text{m}$

Point *D*. $M_D = 472.47 - 472.68 = -0.21 \approx 0$ Checks

The bending moment diagram is shown in Fig. 5.11(d). The shape of this diagram between the ordinates just computed has been based on the condition that the slope of the bending moment diagram at any point is equal to the shear at that point. Just to the right of *A*, the shear is positive, and so is the slope of the bending moment diagram at this point. As we move to the right from *A*, the shear decreases linearly (but remains positive), until it becomes zero at *E*. Therefore, the slope of the bending moment diagram gradually decreases, or becomes less steep (but remains positive), as we move to the right from A, until it becomes zero at *E*. Note that the shear diagram in segment *AE* is linear, but the bending moment diagram in this segment is parabolic, or a second-degree curve, because the bending moment diagram is obtained by integrating the shear diagram (Eq. 5.11). Therefore, the bending moment curve will always be one degree higher than the corresponding shear curve.

We can see from Fig. 5.11(d) that the bending moment becomes locally maximum at point *E*, where the shear changes from positive to the left to negative to the right. As we move to the right from *E*, the shear becomes negative, and it decreases linearly between *E* and *B*. Accordingly, the slope of the bending moment diagram becomes negative to the right of *E*, and it decreases continuously (becomes more steep downward to the right) between *E* and just to the left of *B*. A positive (clockwise) couple acts at *B*, so the bending moment increases abruptly at this point by an amount equal to the magnitude of the moment of the couple. The largest value (global maximum) of the bending moment over the entire length of the beam occurs at just to the right of *B*. (Note that no abrupt change, or discontinuity, occurs in the shear diagram at this point.) Finally, as the shear in segments *BC* and *CD* is constant and negative, the bending moment diagram in these segments consists of straight lines with negative slopes. Ans.

Qualitative Deflected Shape. See Fig. 5.11(e). Ans.

Example 5.8

Draw the shear and bending moment diagrams and the qualitative deflected shape for the beam shown in Fig. 5.12(a).

Solution

Reactions. (See Fig. 5.12(b).)

$$+\rightarrow \sum F_x = 0 \qquad B_x = 0$$

$$+\circlearrowleft \sum M_C = 0$$

$$\frac{1}{2}(3)(12)(24) - B_y(20) + 3(20)(10) - \frac{1}{2}(3)(6)(2) = 0$$

$$B_y = 50.7 \text{ k} \qquad B_y = 50.7 \text{ k} \uparrow$$

$$+\uparrow \sum F_y = 0$$

$$-\frac{1}{2}(3)(12) + 50.7 - 3(20) - \frac{1}{2}(3)(6) + C_y = 0$$

$$C_y = 36.3 \text{ k} \qquad C_y = 36.3 \text{ k} \uparrow$$

continued

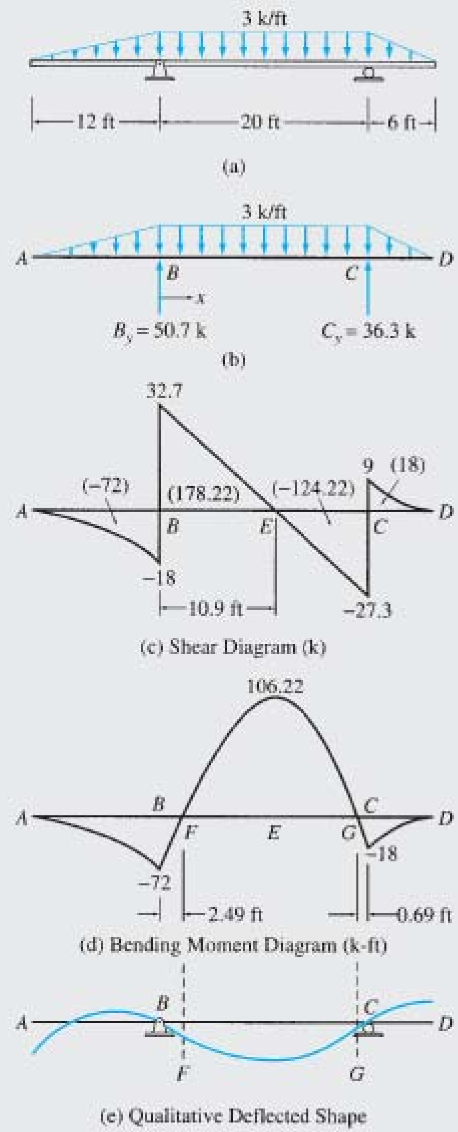


FIG. 5.12

Shear Diagram.

Point *A*.

$$S_A = 0$$

Point *B*.

$$S_{B,L} = 0 - \frac{1}{2}(3)(12) = -18 \text{ k}$$

$$S_{B,R} = -18 + 50.7 = 32.7 \text{ k}$$

continued

Point *C*. $S_{C,L} = 32.7 - 3(20) = -27.3 \text{ k}$

$S_{C,R} = -27.3 + 36.3 = 9 \text{ k}$

Point *D*. $S_D = 9 - \frac{1}{2}(3)(6) = 0$ **Checks**

The shear diagram is shown in Fig. 5.12(c). The shape of the diagram between the ordinates just computed is obtained by applying the condition that the slope of the shear diagram at any point is equal to the load intensity at that point. For example, as the load intensity at *A* is zero, so is the slope of the shear diagram at *A*. Between *A* and *B*, the load intensity is negative and it decreases linearly from zero at *A* to −3 k/ft at *B*. Thus, the slope of the shear diagram is negative in this segment, and it decreases (becomes more steep) continuously from *A* to just to the left of *B*. The rest of the shear diagram is constructed by using similar reasoning. **Ans.**

The point of zero shear, *E*, is located by using the similar triangles forming the shear diagram between *B* and *C*.

To facilitate the construction of the bending moment diagram, the areas of the various segments of the shear diagram have been computed and are shown in parentheses on the shear diagram (Fig. 5.12(c)). It should be noted that the areas of the parabolic spandrels, *AB* and *CD*, can be obtained by using the formula for the area of this shape given in Appendix A.

Bending Moment Diagram.

Point *A*. $M_A = 0$

Point *B*. $M_B = 0 - 72 = -72 \text{ k-ft}$

Point *E*. $M_E = -72 + 178.22 = 106.22 \text{ k-ft}$

Point *C*. $M_C = 106.22 - 124.22 = -18 \text{ k-ft}$

Point *D*. $M_D = -18 + 18 = 0$ **Checks**

The shape of the bending moment diagram between these ordinates is obtained by using the condition that the slope of the bending moment diagram at any point is equal to the shear at that point. The bending moment diagram thus constructed is shown in Fig. 5.12(d).

It can be seen from this figure that the maximum negative bending moment occurs at point *B*, whereas the maximum positive bending moment, which has the largest absolute value over the entire length of the beam, occurs at point *E*. **Ans.**

To locate the points of inflection, *F* and *G*, we set equal to zero the equation for bending moment in segment *BC*, in terms of the distance *x* from the left support point *B* (Fig. 5.12(b)):

$$M = -\left(\frac{1}{2}\right)(3)(12)(4 + x) + 50.7x - 3(x)\left(\frac{x}{2}\right) = 0$$

or

$$-1.5x^2 + 32.7x - 72 = 0$$

from which $x = 2.49$ ft and $x = 19.31$ ft from *B*.

Qualitative Deflected Shape. A qualitative deflected shape of the beam is shown in Fig. 5.12(e). The bending moment is positive in segment *FG*, so the beam is bent concave upward in this region. Conversely, since the bending moment is negative in segments *AF* and *GD*, the beam is bent concave downward in these segments. **Ans.**

Example 5.9

Draw the shear and bending moment diagrams and the qualitative deflected shape for the beam shown in Fig. 5.13(a).

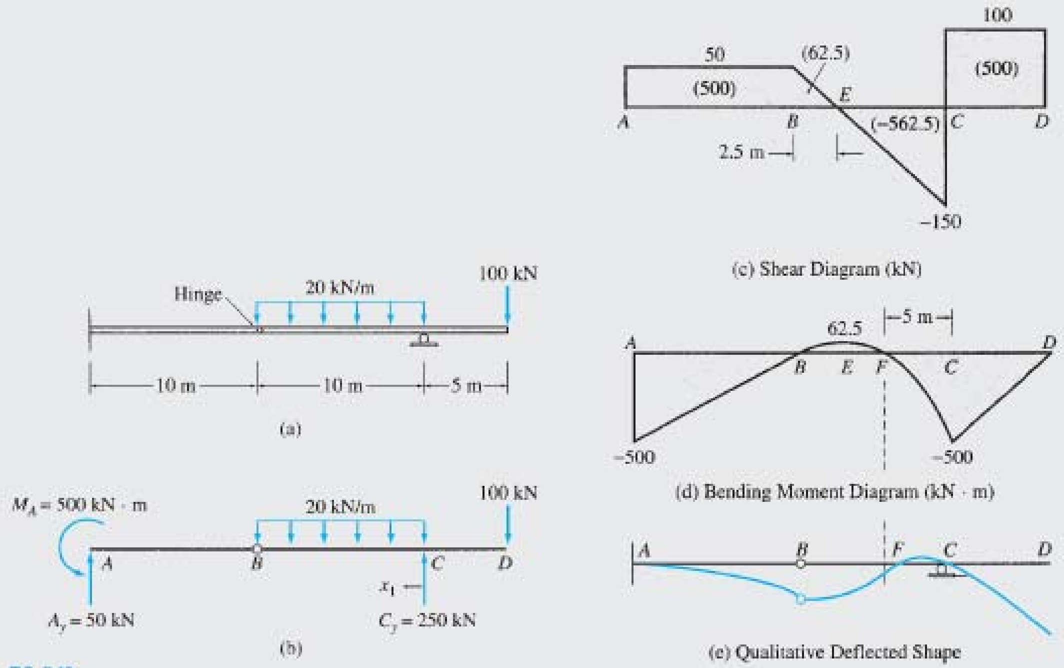


FIG. 5.13

Solution

Reactions. (See Fig. 5.13(b).)

$$+\circlearrowleft \sum M_B^{BD} = 0$$

$$-20(10)(5) + C_y(10) - 100(15) = 0$$

$$C_y = 250 \text{ kN} \qquad C_y = 250 \text{ kN} \uparrow$$

$$+\uparrow \sum F_y = 0$$

$$A_y - 20(10) + 250 - 100 = 0$$

$$A_y = 50 \text{ kN} \qquad A_y = 50 \text{ kN} \uparrow$$

$$+\circlearrowleft \sum M_A = 0$$

$$M_A - 20(10)(15) + 250(20) - 100(25) = 0$$

$$M_A = 500 \text{ kN} \cdot \text{m} \qquad M_A = 500 \text{ kN} \cdot \text{m} \circlearrowleft$$

continued

Shear Diagram.

Point *A*. $S_{A,R} = 50 \text{ kN}$

Point *B*. $S_B = 50 + 0 = 50 \text{ kN}$

Point *C*. $S_{C,L} = 50 - 20(10) = -150 \text{ kN}$

$S_{C,R} = -150 + 250 = 100 \text{ kN}$

Point *D*. $S_{D,L} = 100 + 0 = 100 \text{ kN}$

$S_{D,R} = 100 - 100 = 0$ Checks

The shear diagram is shown in Fig. 5.13(c). Ans.

Bending Moment Diagram.

Point *A*. $M_{A,R} = -500 \text{ kN} \cdot \text{m}$

Point *B*. $M_B = -500 + 500 = 0$

Point *E*. $M_E = 0 + 62.5 = 62.5 \text{ kN} \cdot \text{m}$

Point *C*. $M_C = 62.5 - 562.5 = -500 \text{ kN} \cdot \text{m}$

Point *D*. $M_D = -500 + 500 = 0$ Checks

The bending moment diagram is shown in Fig. 5.13(d). The point of inflection F can be located by setting equal to zero the equation for bending moment in segment BC, in terms of the distance x_1 from the right support point C (Fig. 5.13(b)):

$$M = -100(5 + x_1) + 250x_1 - 20(x_1)\left(\frac{x_1}{2}\right) = 0$$

or

$$-10x_1^2 + 150x_1 - 500 = 0$$

from which $x_1 = 5$ m and $x_1 = 10$ m from C. Note that the solution $x_1 = 10$ m represents the location of the internal hinge at B, at which the bending moment is zero. Thus, the point of inflection F is located at a distance of 5 m to the left of C, as shown in Fig. 5.13(d). Ans.

Qualitative Deflected Shape. A qualitative deflected shape of the beam is shown in Fig. 5.13(e). Note that at the fixed support A, both the deflection and the slope of the beam are zero, whereas at the roller support C, only the deflection is zero, but the slope is not. The internal hinge B does not provide any rotational restraint, so the slope at B can be discontinuous. Ans.

Example 5.10

Draw the shear and bending moment diagrams and the qualitative deflected shape for the beam shown in Fig. 5.14(a).

Solution

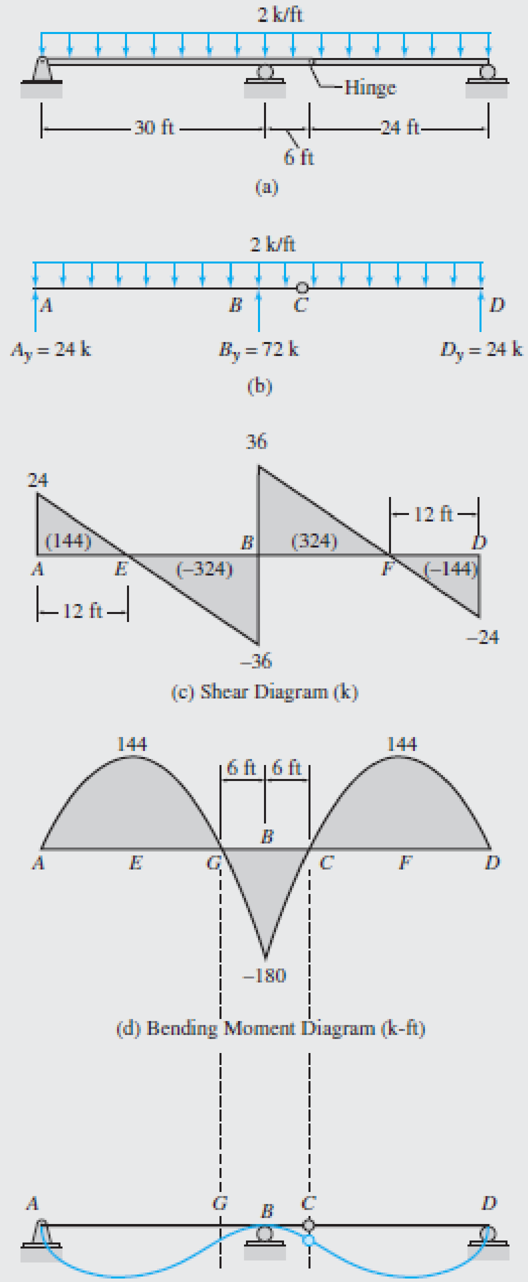
Reactions. (See Fig. 5.14(b).)

$$+\circlearrowleft \sum M_C^{CD} = 0$$

$$D_y(24) - 2(24)(12) = 0$$

$$D_y = 24 \text{ k} \qquad D_y = 24 \text{ k} \uparrow$$

continued



(c) Shear Diagram (k)

(d) Bending Moment Diagram (k-ft)

(e) Qualitative Deflected Shape

FIG. 5.14

continued

$$+\circlearrowleft \sum M_A = 0$$

$$24(60) + B_y(30) - 2(60)(30) = 0$$

$$B_y = 72 \text{ k} \qquad B_y = 72 \text{ k} \uparrow$$

$$+\uparrow \sum F_y = 0$$

$$A_y - 2(60) + 72 + 24 = 0$$

$$A_y = 24 \text{ k} \qquad A_y = 24 \text{ k} \uparrow$$

Shear Diagram.

Point *A*. $S_{A,R} = 24$ k

Point *B*. $S_{B,L} = 24 - 2(30) = -36$ k

$S_{B,R} = -36 + 72 = 36$ k

Point *D*. $S_{D,L} = 36 - 2(30) = -24$ k

$S_{D,R} = -24 + 24 = 0$ Checks

The shear diagram is shown in Fig. 5.14(c). Ans.

Bending Moment Diagram.

Point *A*. $M_A = 0$

Point *E*. $M_E = 0 + 144 = 144$ k-ft

Point *B*. $M_B = 144 - 324 = -180$ k-ft

Point *F*. $M_F = -180 + 324 = 144$ k-ft

Point *D*. $M_D = 144 - 144 = 0$ Checks

The bending moment diagram is shown in Fig. 5.14(d). Ans.

Qualitative Deflected Shape. See Fig. 5.14(e). Ans.

Example 5.11

Draw the shear and bending moment diagrams and the qualitative deflected shape for the statically indeterminate beam shown in Fig. 5.15. The support reactions, determined by using the procedures for the analysis of statically indeterminate beams (presented in Part Three of this text), are given in Fig. 5.15(a).

Solution

Regardless of whether a beam is statically determinate or indeterminate, once the support reactions have been determined, the procedure for constructing the shear and bending moment diagrams remains the same. The shear and bending moment diagrams for the given statically indeterminate beam are shown in Fig. 5.15(b) and (c), respectively, and a qualitative deflected shape of the beam is shown in Fig. 5.15(d).

continued

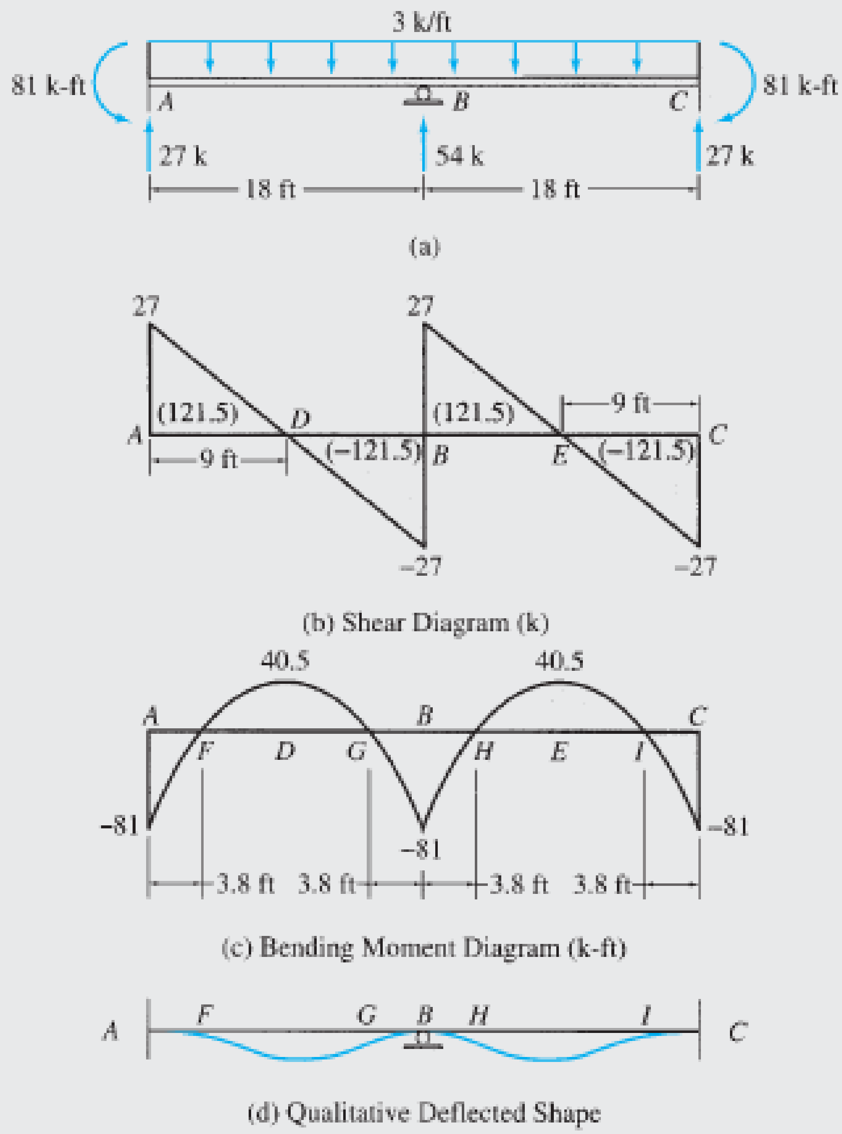

3 k/ft
81 k-ft
81 k-ft
A
B
C
27 k
54 k
27 k
18 ft
18 ft
(a)
27
27
(121.5)
(121.5)
9 ft
D
(−121.5)
(−121.5)
9 ft
A
B
E
C
−27
−27
(b) Shear Diagram (k)
40.5
40.5
A
B
C
F
D
G
H
E
I
−81
−81
−81
3.8 ft
3.8 ft
3.8 ft
3.8 ft
(c) Bending Moment Diagram (k-ft)
A
F
G
B
H
I
C
(d) Qualitative Deflected Shape

FIG. 5.15

Example 5.13

Draw the shear, bending moment, and axial force diagrams and the qualitative deflected shape for the frame shown in Fig. 5.23(a).

Solution

Static Determinacy. $m = 3$, $j = 4$, $r = 3$, and $e_c = 0$. Because $3m + r = 3j + e_c$ and the frame is geometrically stable, it is statically determinate.

Reactions. Considering the equilibrium of the entire frame (Fig. 5.23(b)), we observe that in order to satisfy $\sum F_X = 0$, the reaction component A_X must act to the left with a magnitude of 18 k to balance the horizontal load of 18 k to the right. Thus,

$$A_X = -18 \text{ k} \qquad A_X = 18 \text{ k} \leftarrow$$

We compute the remaining two reactions by applying the two equilibrium equations as follows:

$$+\circlearrowleft \sum M_A = 0 \qquad -18(20) - 2(30)(15) + D_Y(30) = 0 \qquad D_Y = 42 \text{ k} \uparrow$$

$$+\uparrow \sum F_Y = 0 \qquad A_Y - 2(30) + 42 = 0 \qquad A_Y = 18 \text{ k} \uparrow$$

Member End Forces. The free-body diagrams of all the members and joints of the frame are shown in Fig. 5.23(c). We can begin the computation of internal forces either at joint A or at joint D, both of which have only three unknowns.

Joint A. Beginning with joint A, we can see from its free-body diagram that in order to satisfy $\sum F_X = 0$, A_X^{AB} must act to the right with a magnitude of 18 k to balance the horizontal reaction of 18 k to the left. Thus,

$$A_X^{AB} = -18 \text{ k}$$

continued

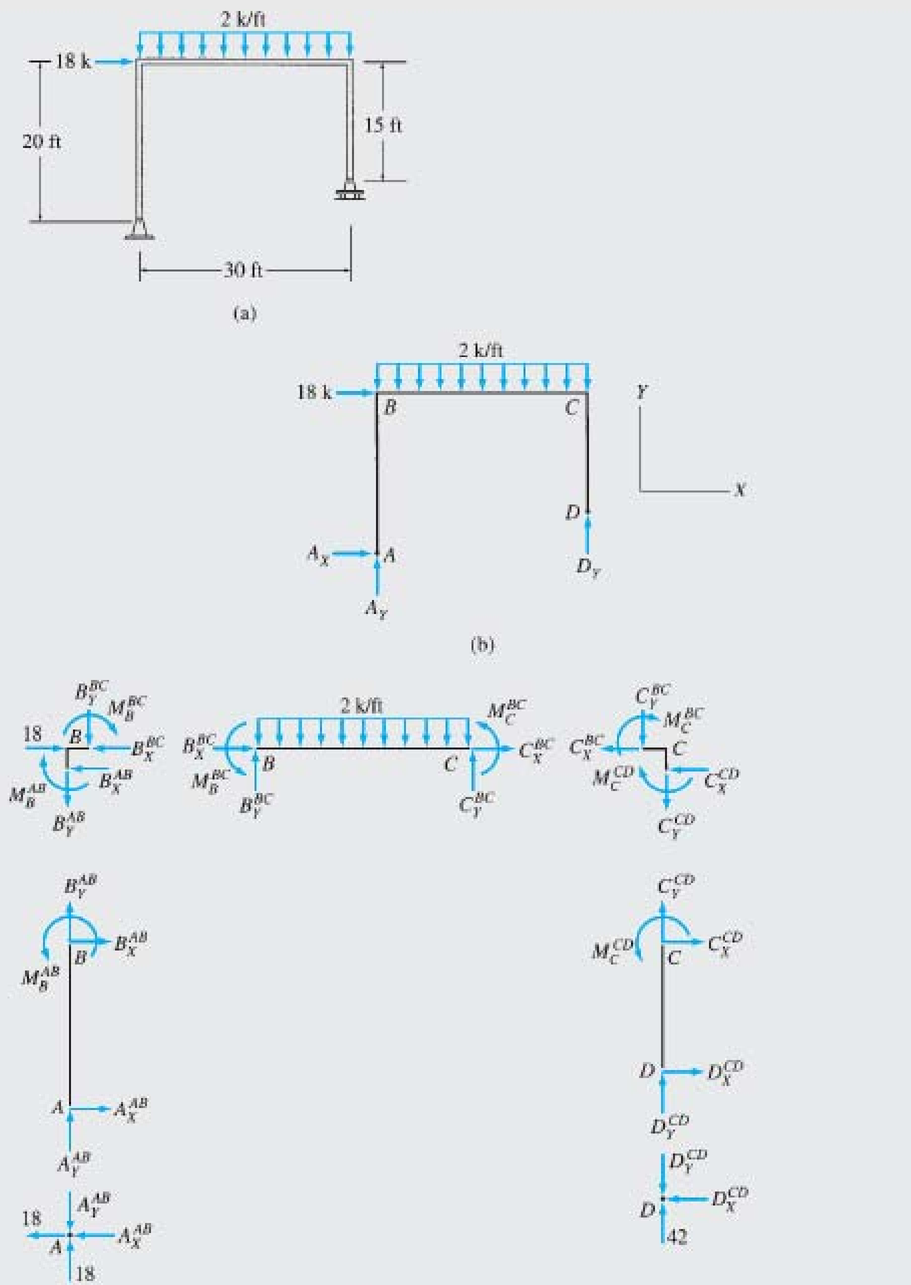


FIG. 5.23

continued

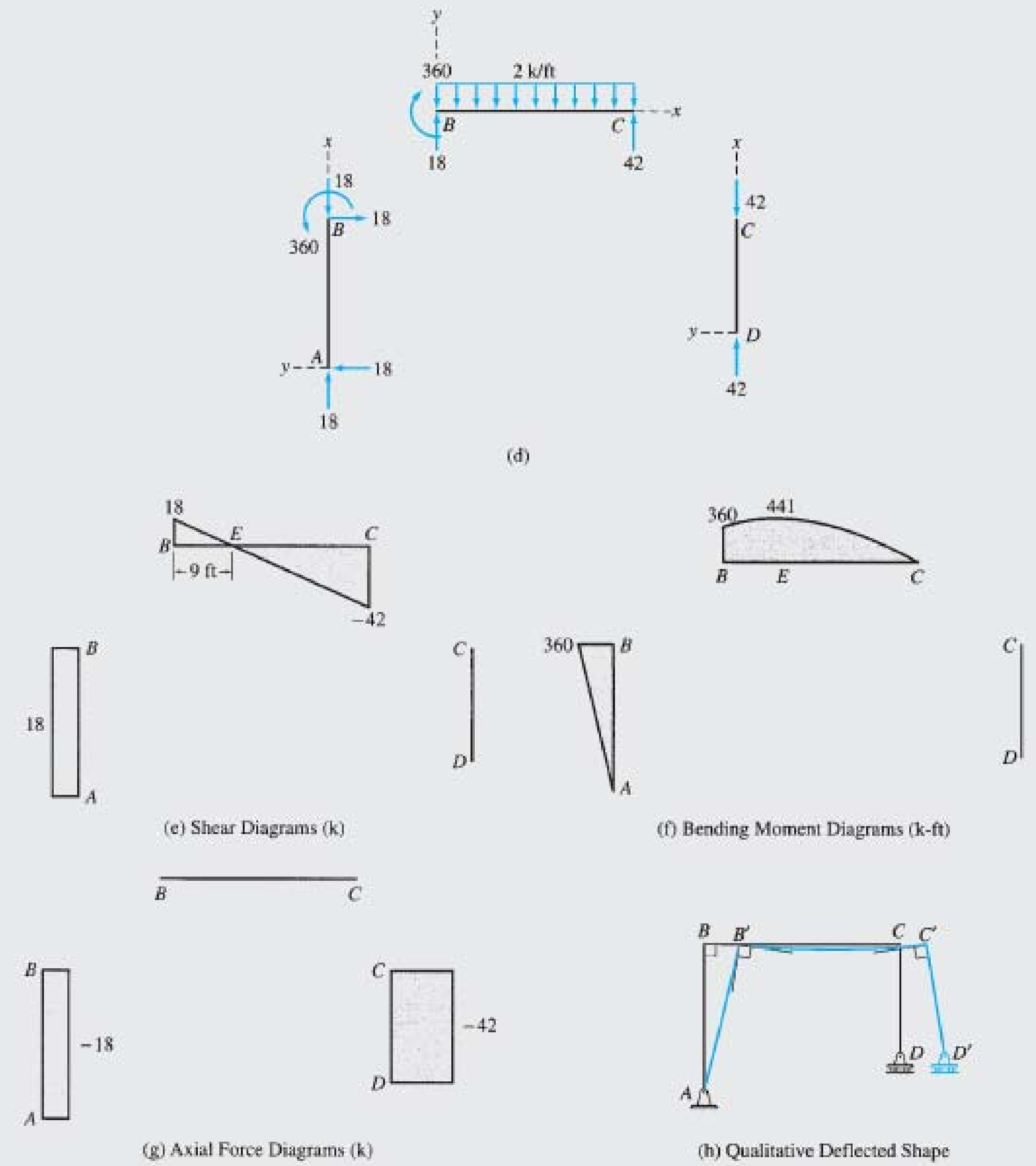


FIG. 5.23 (contd.)

continued

Similarly, by applying $\sum F_Y = 0$, we obtain

$$A_Y^{AB} = 18 \text{ k}$$

Member AB. With the magnitudes of A_X^{AB} and A_Y^{AB} now known, member AB has three unknowns, B_X^{AB}, B_Y^{AB}, and M_B^{AB}, which can be determined by applying $\sum F_X = 0$, $\sum F_Y = 0$, and $\sum M_A = 0$. Thus,

$$B_X^{AB} = 18 \text{ k} \qquad B_Y^{AB} = -18 \text{ k} \qquad M_B^{AB} = 360 \text{ k-ft}$$

Joint B. Proceeding next to joint B and considering its equilibrium, we obtain

$$B_X^{BC} = 0 \qquad B_Y^{BC} = 18 \text{ k} \qquad M_B^{BC} = -360 \text{ k-ft}$$

Member BC. Next, considering the equilibrium of member BC, we write

$$+ \rightarrow \sum F_X = 0 \qquad C_X^{BC} = 0$$

$$+ \uparrow \sum F_Y = 0 \qquad 18 - 2(30) + C_Y^{BC} = 0 \qquad C_Y^{BC} = 42 \text{ k}$$

$$+ \circlearrowleft \sum M_B = 0 \qquad -360 - 2(30)(15) + 42(30) + M_C^{BC} = 0 \qquad M_C^{BC} = 0$$

Joint C. Applying the three equilibrium equations, we obtain

$$C_X^{CD} = 0 \qquad C_Y^{CD} = -42 \text{ k} \qquad M_C^{CD} = 0$$

Member CD. Applying $\sum F_X = 0$ and $\sum F_Y = 0$ in order, we obtain

$$D_X^{CD} = 0 \qquad D_Y^{CD} = 42 \text{ k}$$

Since all unknown forces and moments have been determined, we check our computations by using the third equilibrium equations for member CD.

$$+ \circlearrowleft \sum M_D = 0$$ Checks

Joint D. (Checking computations)

$$+ \rightarrow \sum F_X = 0$$ Checks

$$+ \uparrow \sum F_Y = 0 \qquad 42 - 42 = 0$$ Checks

Shear Diagrams. The xy coordinate systems selected for the three members of the frame are shown in Fig. 5.23(d), and the shear diagrams for the members constructed by using the procedure described in Section 5.4 are depicted in Fig. 5.23(e). **Ans.**

Bending Moment Diagrams. The bending moment diagrams for the three members of the frame are shown in Fig. 5.23(f).

Axial Force Diagrams. From the free-body diagram of member AB in Fig. 5.23(d), we observe that the axial force throughout the length of this member is compressive, with a constant magnitude of 18 k. Therefore, the axial force diagram for this member is a straight line parallel to the x axis at a value of -18 k, as shown in Fig. 5.23(g). Similarly, it can be seen from Fig. 5.23(d) that the axial forces in members BC and CD are also constant, with magnitudes of 0 and -42 k, respectively. The axial force diagrams thus constructed for these members are shown in Fig. 5.23(g). **Ans.**

Qualitative Deflected Shape. From the bending moment diagrams of the members of the frame (Fig. 5.23(f)), we observe that the members AB and BC bend concave to the left and concave upward, respectively. As no bending moment develops in member CD, it does not bend but remains straight. A qualitative deflected shape of the frame obtained by connecting the deflected shapes of the three members at the joints is shown in Fig. 5.23(h). As this figure indicates, the deflection of the frame at support A is zero. Due to the horizontal load at B, joint B deflects to the right to B'. Since the axial deformations of members are neglected and bending deformations are assumed to be small, joint B deflects only in the horizontal direction, and joint C deflects by the same amount as joint B; that is, $BB' = CC'$. Note that the curvatures of the members are consistent with their bending moment diagrams and that the original 90° angles between members at the rigid joints B and C have been maintained. **Ans.**

Example 5.14

Draw the shear, bending moment, and axial force diagrams and the qualitative deflected shape for the frame shown in Fig. 5.24(a).

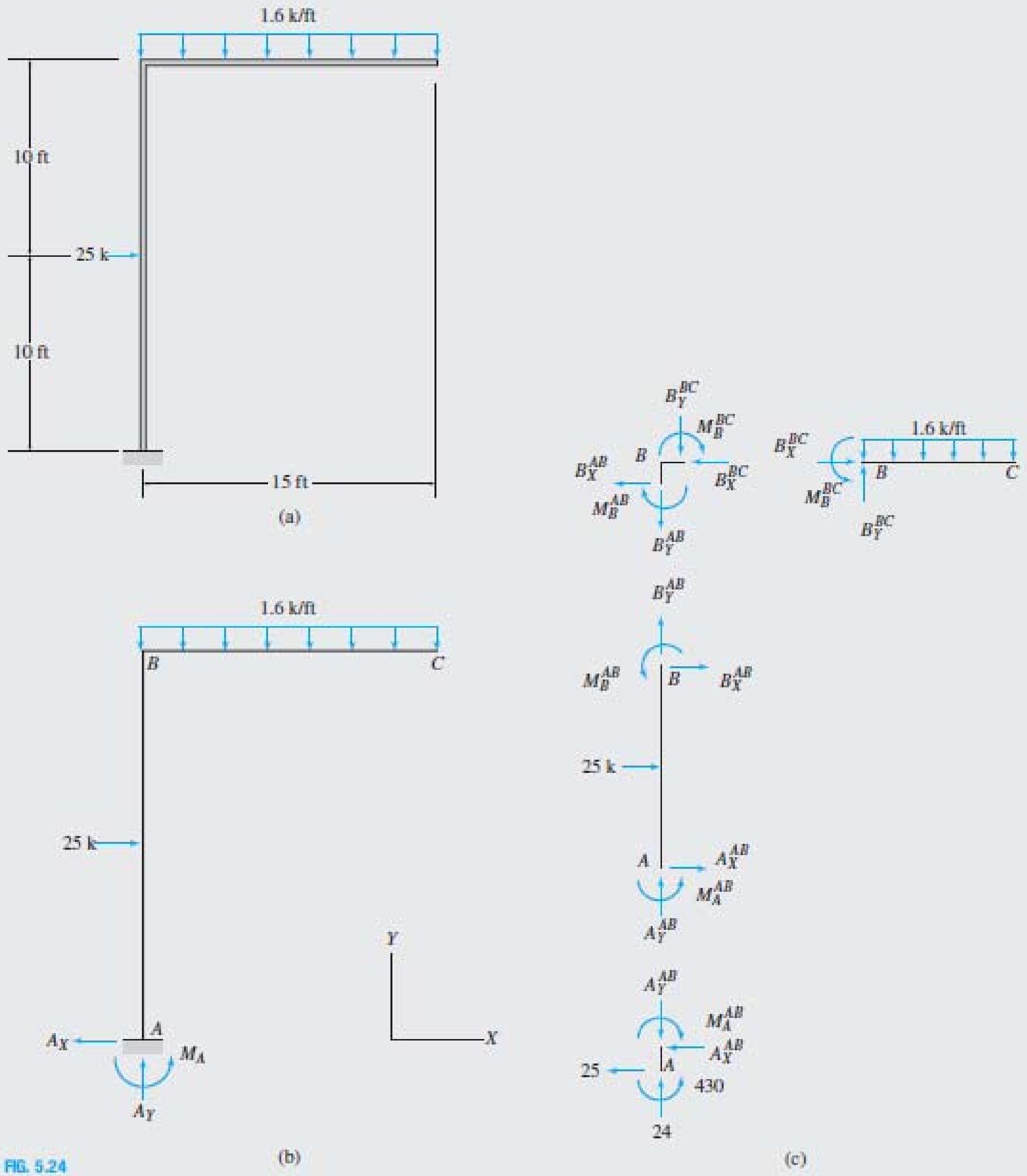
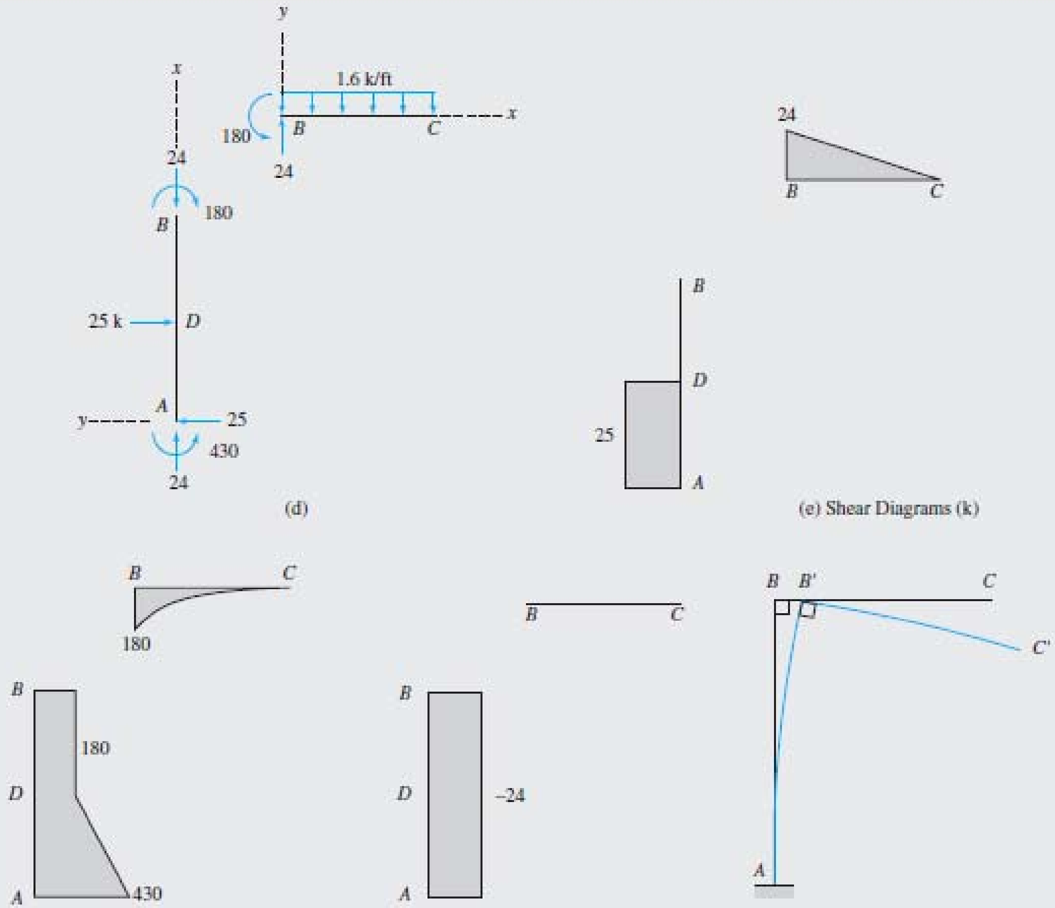


FIG. 5.24

continued



(d)

(e) Shear Diagrams (k)

(f) Bending Moment Diagrams (k-ft)

(g) Axial Force Diagrams (k)

(h) Qualitative Deflected Shape

FIG. 5.24 (contd.)

Solution

Static Determinacy. $m = 2$, $j = 3$, $r = 3$, and $e_c = 0$. Because $3m + r = 3j + e_c$ and the frame is geometrically stable, it is statically determinate.

Reactions. (See Fig. 5.24(b).)

$$+ \rightarrow \sum F_x = 0$$

$$-A_x + 25 = 0 \qquad A_x = 25 \text{ k} \leftarrow$$

$$+ \uparrow \sum F_y = 0$$

$$A_y - 1.6(15) = 0 \qquad A_y = 24 \text{ k} \uparrow$$

$$+ \circlearrowleft \sum M_A = 0$$

$$M_A - 25(10) - 1.6(15)(7.5) = 0 \qquad M_A = 430 \text{ k-ft} \circlearrowleft$$

continued

Member End Forces. (See Fig. 5.24(c).)

Joint *A*. By applying the equilibrium equations $\sum F_X = 0$, $\sum F_Y = 0$, and $\sum M_A = 0$, we obtain

$$A_X^{AB} = -25\text{ k} \qquad A_Y^{AB} = 24\text{ k} \qquad M_A^{AB} = 430\text{ k-ft}$$

Member *AB*. Next, considering the equilibrium of member *AB*, we write

$$+\rightarrow \sum F_X = 0 \qquad -25 + 25 + B_X^{AB} = 0 \qquad B_X^{AB} = 0$$

$$+\uparrow \sum F_Y = 0 \qquad 24 + B_Y^{AB} = 0 \qquad B_Y^{AB} = -24\text{ k}$$

$$+\circlearrowleft \sum M_B = 0 \qquad 430 - 25(10) + M_B^{AB} = 0 \qquad M_B^{AB} = -180\text{ k-ft}$$

Joint *B*. Applying the three equations of equilibrium, we obtain

$$B_X^{BC} = 0 \qquad B_Y^{BC} = 24\text{ k} \qquad M_B^{BC} = 180\text{ k-ft}$$

Member *BC*. (Checking computations.)

$$+\rightarrow \sum F_X = 0 \qquad \text{Checks}$$

$$+\uparrow \sum F_Y = 0 \qquad 24 - 1.6(15) = 0 \qquad \text{Checks}$$

$$+\circlearrowleft \sum M_B = 0 \qquad 180 - 1.6(15)(7.5) = 0 \qquad \text{Checks}$$

The member end forces are shown in Fig. 5.24(d).

Shear Diagrams. See Fig. 5.24(e). Ans.

Bending Moment Diagrams. See Fig. 5.24(f). Ans.

Axial Force Diagrams. See Fig. 5.24(g). Ans.

Qualitative Deflected Shape. See Fig. 5.24(h). Ans.

Example 5.15

A gable frame is subjected to a snow loading, as shown in Fig. 5.25(a). Draw the shear, bending moment, and axial force diagrams and the qualitative deflected shape for the frame.

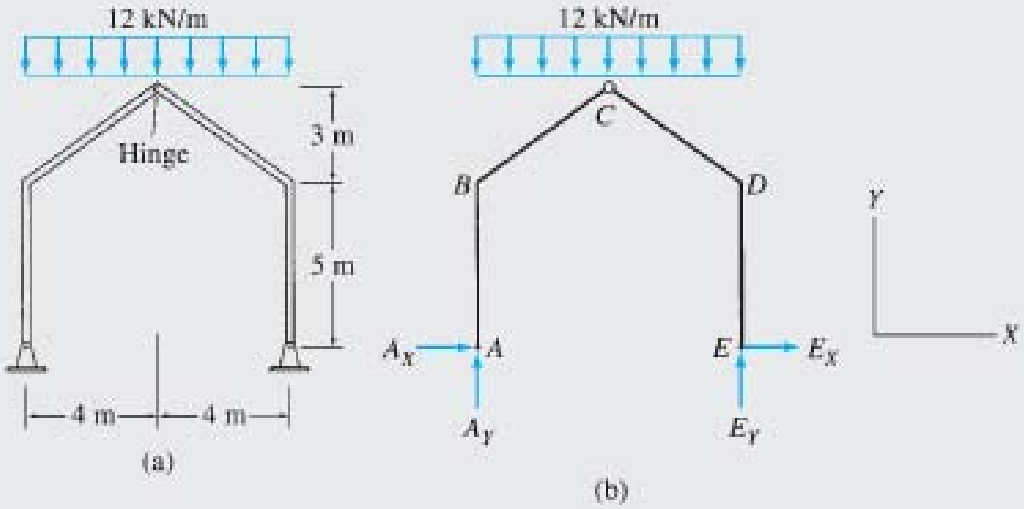


FIG. 5.25

continued

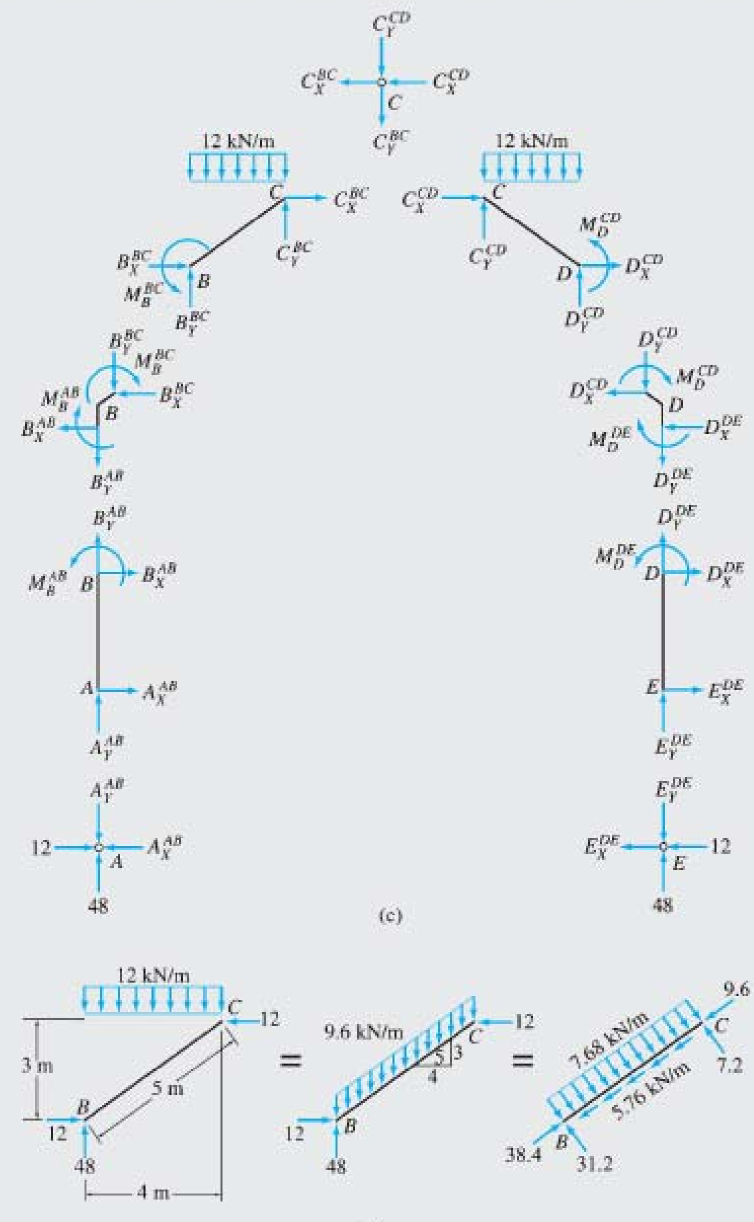
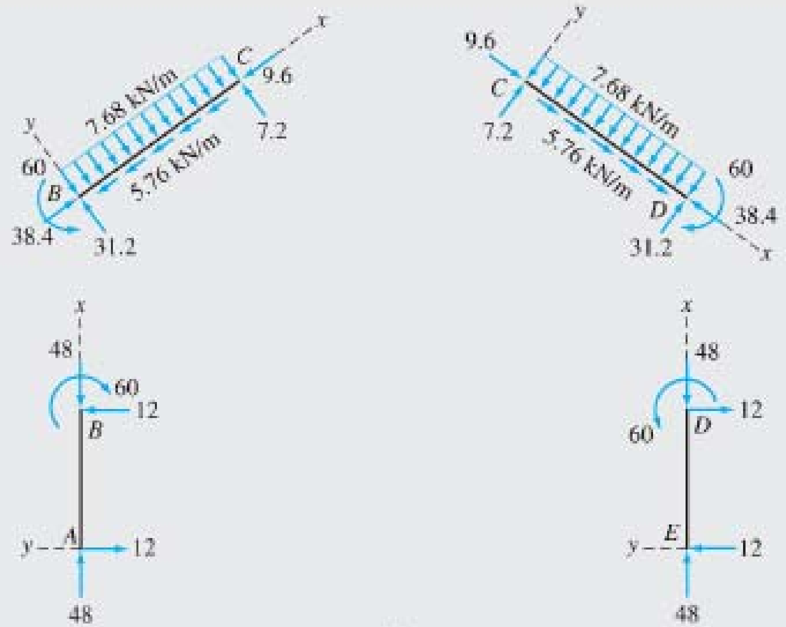
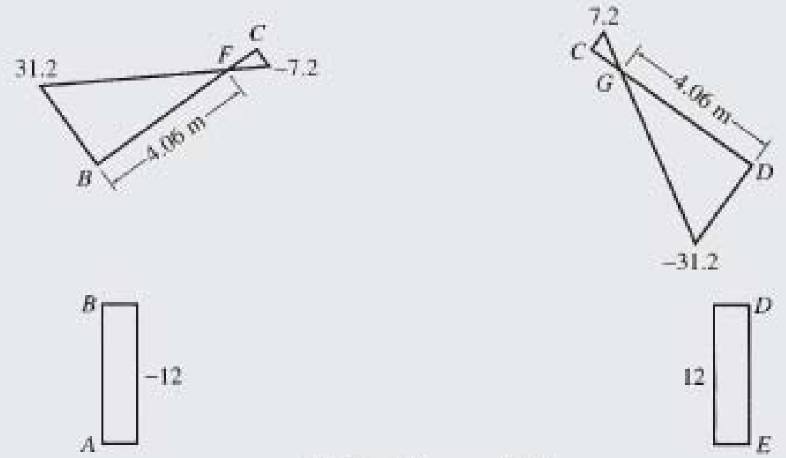


FIG. 5.25 (contd.)

continued



(e)



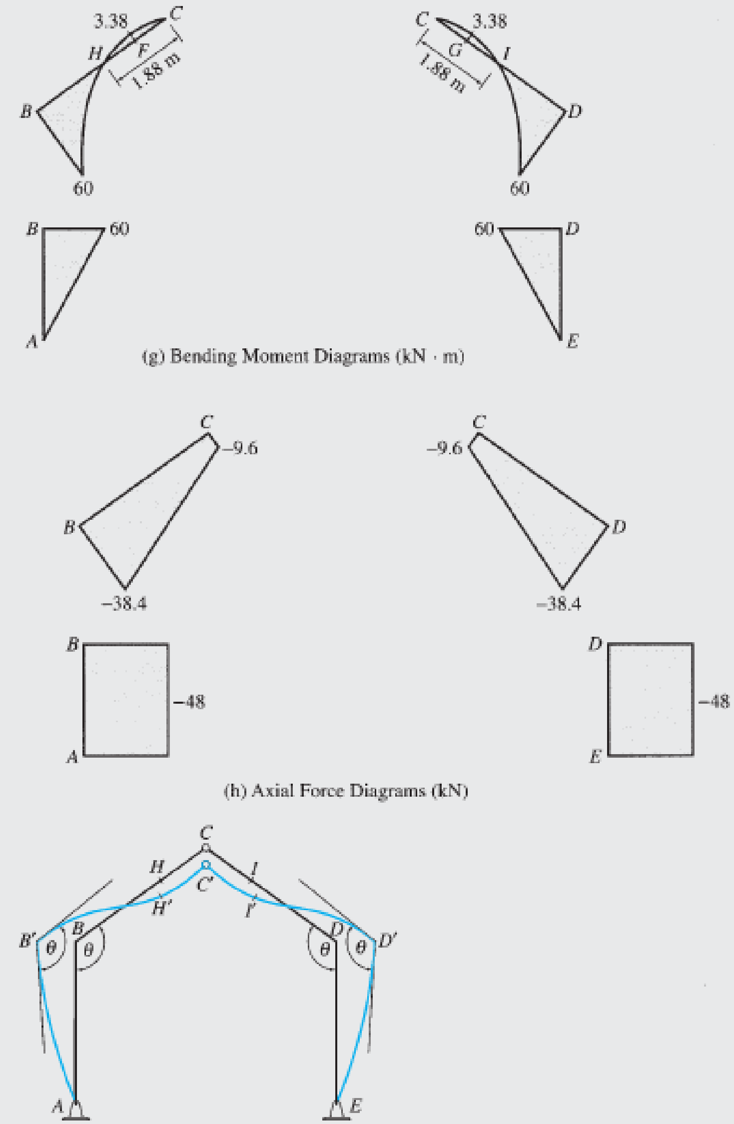
(f) Shear Diagrams (kN)

FIG. 5.25 (contd.)

continued

Solution

Static Determinacy. $m = 4$, $j = 5$, $r = 4$, and $e_c = 1$. Because $3m + r = 3j + e_c$ and the frame is geometrically stable, it is statically determinate.



(g) Bending Moment Diagrams (kN · m)

(h) Axial Force Diagrams (kN)

(i) Qualitative Deflected Shape

FIG. 5.25 (contd.)

continued

Reactions. (See Fig. 5.25(b).)

$$+\circlearrowleft \sum M_E = 0$$

$$-A_Y(8) + 12(8)(4) = 0 \qquad A_Y = 48 \text{ kN} \uparrow$$

$$+\uparrow \sum F_Y = 0$$

$$48 - 12(8) + E_Y = 0 \qquad E_Y = 48 \text{ kN} \uparrow$$

$$+\circlearrowleft \sum M_C^{AC} = 0$$

$$A_X(8) - 48(4) + 12(4)(2) = 0 \qquad A_X = 12 \text{ kN} \rightarrow$$

$$+\rightarrow \sum F_X = 0$$

$$12 + E_X = 0$$

$$E_X = -12 \text{ kN} \qquad E_X = 12 \text{ kN} \leftarrow$$

Member End Forces. (See Fig. 5.25(c).)

Joint *A*. By applying the equations of equilibrium $\sum F_X = 0$ and $\sum F_Y = 0$, we obtain

$$A_X^{AB} = 12 \text{ kN} \qquad A_Y^{AB} = 48 \text{ kN}$$

Member *AB*. Considering the equilibrium of member AB, we obtain

$$B_X^{AB} = -12 \text{ kN} \qquad B_Y^{AB} = -48 \text{ kN} \qquad M_B^{AB} = -60 \text{ kN} \cdot \text{m}$$

Joint *B*. Applying the three equilibrium equations, we obtain

$$B_X^{BC} = 12 \text{ kN} \qquad B_Y^{BC} = 48 \text{ kN} \qquad M_B^{BC} = 60 \text{ kN} \cdot \text{m}$$

Member *BC*.

$$+\rightarrow \sum F_X = 0 \qquad C_X^{BC} = -12 \text{ kN}$$

$$+\uparrow \sum F_Y = 0$$

$$48 - 12(4) + C_Y^{BC} = 0 \qquad C_Y^{BC} = 0$$

$$+\circlearrowleft \sum M_B = 0$$

$$60 - 12(4)(2) + 12(3) = 0 \qquad \text{Checks}$$

Joint *C*. Considering the equilibrium of joint C, we determine

$$C_X^{CD} = 12 \text{ kN} \qquad C_Y^{CD} = 0$$

Member *CD*.

$$+\rightarrow \sum F_X = 0 \qquad D_X^{CD} = -12 \text{ kN}$$

$$+\uparrow \sum F_Y = 0$$

$$-12(4) + D_Y^{CD} = 0 \qquad D_Y^{CD} = 48 \text{ kN}$$

$$+\circlearrowleft \sum M_D = 0$$

$$-12(3) + 12(4)(2) + M_D^{CD} = 0 \qquad M_D^{CD} = -60 \text{ kN} \cdot \text{m}$$

Joint *D*. Applying the three equilibrium equations, we obtain

$$D_X^{DE} = 12 \text{ kN} \qquad D_Y^{DE} = -48 \text{ kN} \qquad M_D^{DE} = 60 \text{ kN} \cdot \text{m}$$

continued

Member *DE*.

$$\stackrel{+}{\rightarrow} \sum F_X = 0 \qquad E_X^{DE} = -12 \text{ kN}$$

$$+\uparrow \sum F_Y = 0 \qquad E_Y^{DE} = 48 \text{ kN}$$

$$+\circlearrowleft \sum M_E = 0$$

$$60 - 12(5) = 0 \qquad \text{Checks}$$

Joint *E*.

$$\stackrel{+}{\rightarrow} \sum F_X = 0 \qquad -12 + 12 = 0 \qquad \text{Checks}$$

$$+\uparrow \sum F_Y = 0 \qquad 48 - 48 = 0 \qquad \text{Checks}$$

Distributed Loads on Inclined Members *BC* and *CD*. As the 12-kN/m snow loading is specified per horizontal meter, it is necessary to resolve it into components parallel and perpendicular to the directions of members *BC* and *CD*. Consider, for example, member *BC*, as shown in Fig. 5.25(d). The total vertical load acting on this member is (12 kN/m)(4 m) = 48 kN. Dividing this total vertical load by the length of the member, we obtain the intensity of the vertical distributed load per meter along the inclined length of the member as 48/5 = 9.6 kN/m. The components of this vertical distributed load in the directions parallel and perpendicular to the axis of the member are (3/5)(9.6) = 5.76 kN/m and (4/5)(9.6) = 7.68 kN/m, respectively, as shown in Fig. 5.25(d). The distributed loading for member *CD* is computed similarly and is shown in Fig. 5.25(e).

Shear and Bending Moment Diagrams. See Fig. 5.25(f) and (g). Ans.

Axial Force Diagrams. The equations for axial force in the members of the frame are:

Member *AB* $Q = -48$

Member *BC* $Q = -38.4 + 5.76x$

Member *CD* $Q = -9.6 - 5.76x$

Member *DE* $Q = -48$

The axial force diagrams are shown in Fig. 5.25(h). Ans.

Qualitative Deflected Shape. See Fig. 5.25(i). Ans.