تحلیل خرپا ها

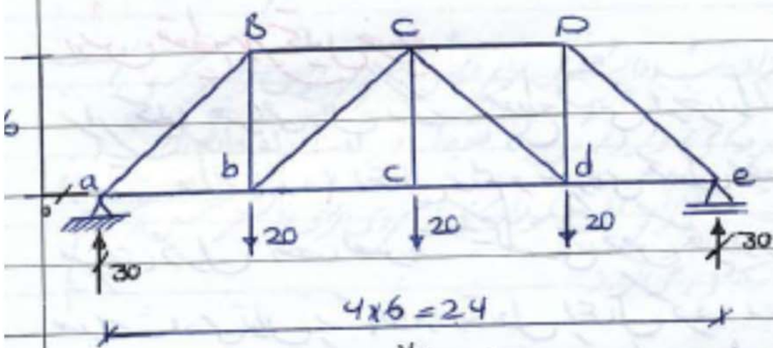
روش گره برای تحلیل خرپاها

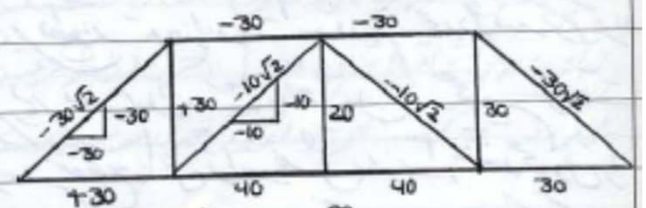
در روش گره برای تحلیل خرپا، ابتدا واکنش های تکیه گاهی محاسبه می گردد سپس گره های خرپا بصورت مجزا آزاد شده و نمودار آزاد آن ترسیم می گردد. در این نمودار آزاد نیروهای معلوم در جهت واقعی و نیروهای مجهول بعضا بصورت کششی نمایش داده می شوند. سپس معادلات تعادل را برای این گره اعمال می کنیم. با توجه به اینکه نیروها متقارب هستند دو معادله تعادل بیشتر نخواهیم داشت.

$$\sum F_x = 0 \qquad \sum F_y = 0$$

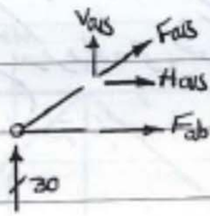
بنابراین باید گره ای را آزاد نماییم که دو مجهول بیشتر نداشته باشد. همچنین توصیه می شود با ترتیب مناسب این توانایی ایجاد گردد که طراح بدون نوشتن معادلات تعادل اقدام به تعیین نیروها نمایند. در حالات متعددی این موضوع امکان پذیر می باشد.

مثال: خرپای نشان داده شده در شکل را تحلیل نمایید.





گره a:



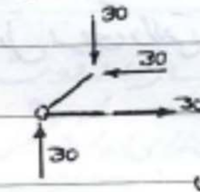
$\Sigma F_y = 0 \quad +30 + V_{ab} = 0 \rightarrow V_{ab} = -30\,ton$

$H_{ab} = -30 \quad F_{ab} = -30\sqrt{2}$ فشاری

$\Sigma F_x = 0 \rightarrow F_{ab} + H_{ab} = 0$

$\rightarrow F_{ab} - 30 = 0 \rightarrow F_{ab} = 30$ کششی

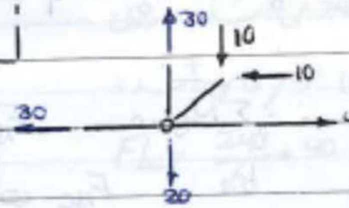
گره a:



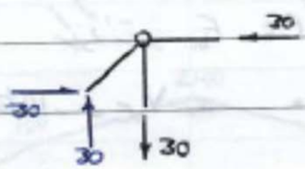
روش استاتیک ذهنی

* روش استاتیک ذهنی وقتی قابل استفاده است که در گره ۲ معادله ۲ مجهولی نداشته باشیم.

گره b:



گره B:



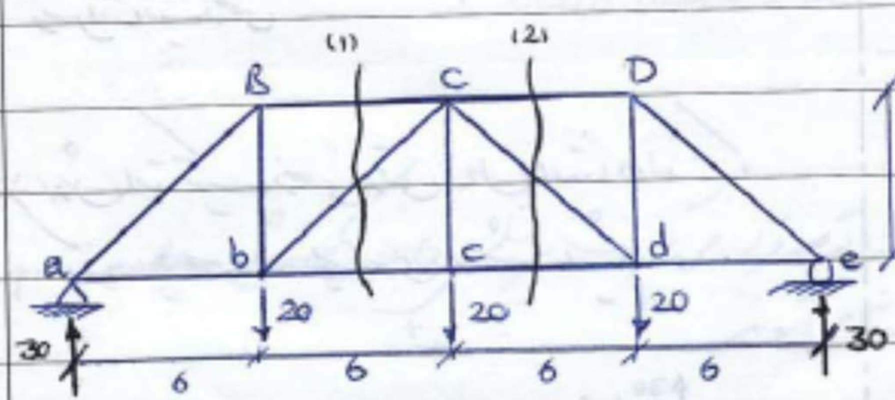
گره c (کوپلیک):

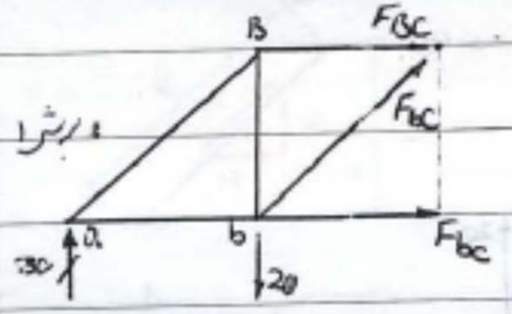
* بقیه اعضا را توسط تقارن می‌توان بدست آورد.

روش مقطع برای تحلیل خرپاها

برای تحلیل خرپاها می‌توان به کمک مقطع قسمتی از خرپا را از سایر قسمت‌ها جدا نماییم، و معادلات تعادل را بر آن (قسمت جدا شده) اعمال نماییم و نیروهای مجهول اعضای قطع شده را بدست آوریم. مجدداً قید می‌گردد نیروهای مجهول اعضا به صورت کششی فرض گردد و نیروهای معلوم در جهت واقعی‌شان. همچنین بر قسمت جدا شده می‌توان به ۳ معادله تعادل اعمال کرد. بنابراین مقطع‌ها نمی‌توانند بیش از ۳ عضو مجهول را قطع نمایند. همچنین تذکر داده می‌شود که در هنگام نوشتن معادلات تعادل از معادلاتی استفاده کنیم که بیش از یک مجهول نداشته باشد.

مثال: خرپای نشان داده شده در مثال قبل را به روش مقطع تحلیل نمایید.

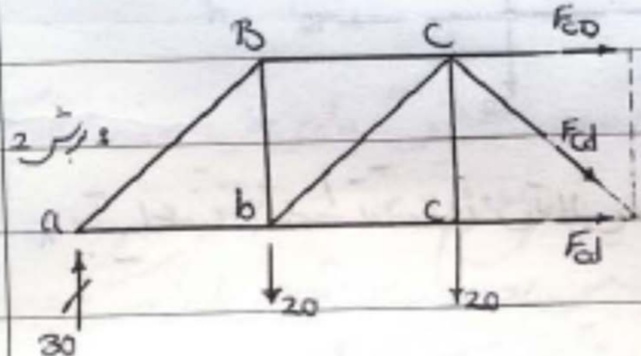




$\sum M_b = 0 \quad -F_{BC} \times 6 - 30 \times 6 = 0 \rightarrow F_{BC} = -30 \text{ ton}$

b : F_{BC} مرکز گشتاور — نقطه C : F_{bc} مرکز گشتاور

$\sum F_y = 0 \rightarrow +30 - 20 + V_{bC} = 0 \rightarrow V_{bC} = -10 \text{ ton}$



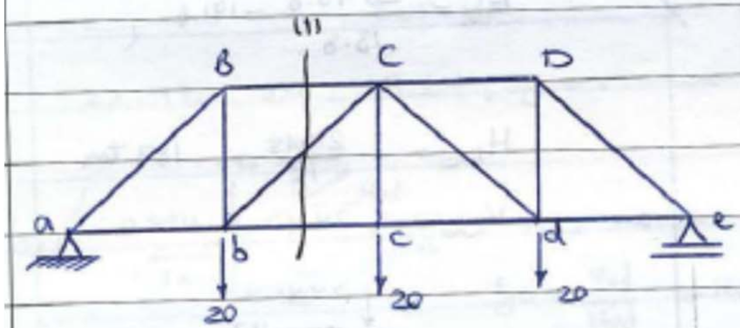
$\sum M_{C} = 0 \quad F_{cd} \times 6 + 20 \times 6 - 30 \times 12 = 0 \rightarrow F_{cd} = 40 \text{ ton}$ (نقطه C)

$\sum F_y = 0 \rightarrow +30 - 20 - 20 + V_{Cd} = 0 \rightarrow V_{Cd} = 10 \text{ ton}$

روش نیروی برشی و لنگر خمشی

توضیحاتی که در قسمت قبل برای روش مقاطع داده شد، پایه تحلیل برای روش نیروی برشی و لنگر خمشی است.
این روش در خرپاهایی قابل استفاده است که نیروهای وارد بر آن فقط در امتداد قائم باشد.
در این روش ابتدا تیری معادل خرپا در نظر گرفته شده و برای آن نمودارهای نیروی برشی و لنگر خمشی رسم می‌گردد. به کمک این دو نمودار خرپا به سهولت تحلیل می‌گردد.

مثال: خرپای نشان داده شده در شکل مقابل را با روش نیروی برشی و لنگر خمشی تحلیل کنید.

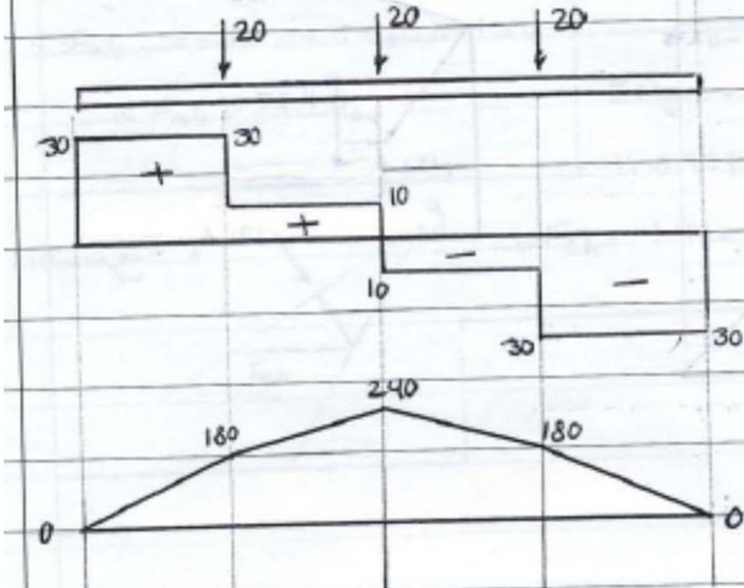


$$F_{bc} = \frac{240}{6} = 40$$

$$F_{BC} = \frac{180}{6} = -30$$

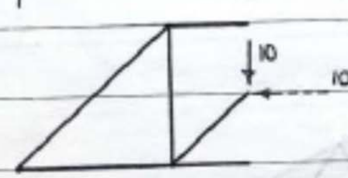
نقطه C (بزرگ) مرکز گشتاور F_{bc} می باشد و چون در نمودار لنگر خمشی دارای لنگر 240 می باشد و بازوی F_{bc} نسبت به نقطه C (بزرگ) برابر 6 می باشد، پس:

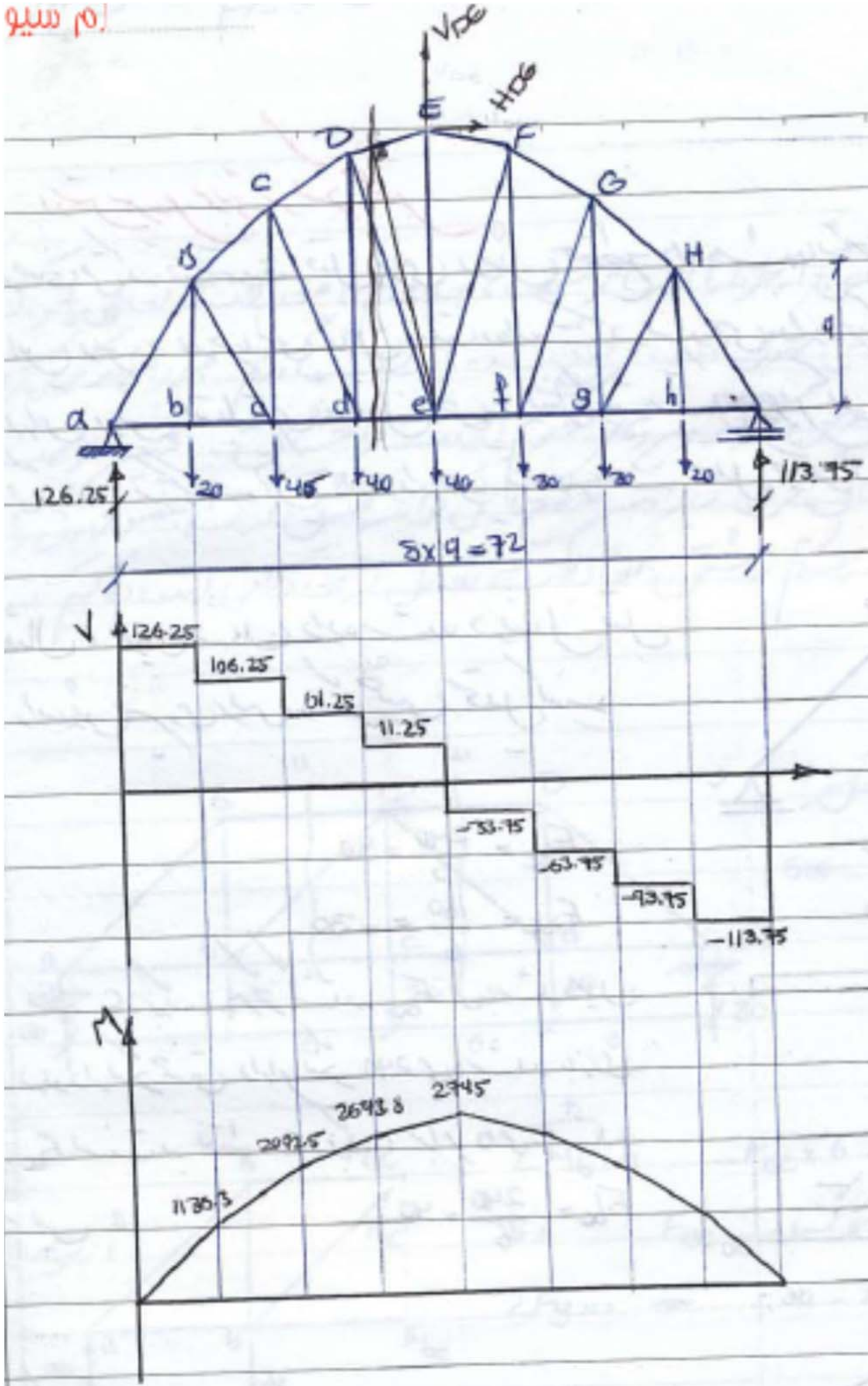
$$F_{bc} = \frac{240}{6} = 40$$



* لنگر مثبت تار پایین را می کشد و تار بالا را می فشارد.

با استفاده از نمودار نیروی برشی می توان مولفه قائم کلیه اعضای قطری را بدست آورد. مقطع (1) را که در آن عضو قطری Cb قطع شده را در نظر می گیریم. طبق قرارداد نیروی برشی، چون در مقطع (1) نیروی برشی مثبت است، پس جهت نیروی برشی مقاوم در مقطع قسمت سمت چپ به سمت پایین می باشد. بنابراین مولفه قائم نیروی عضوی Cb به سمت پایین و مساوی 10 تن است.



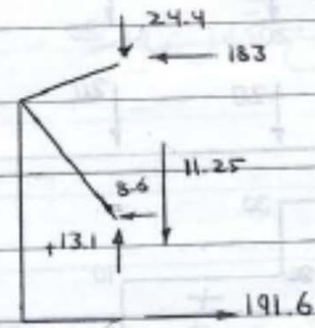


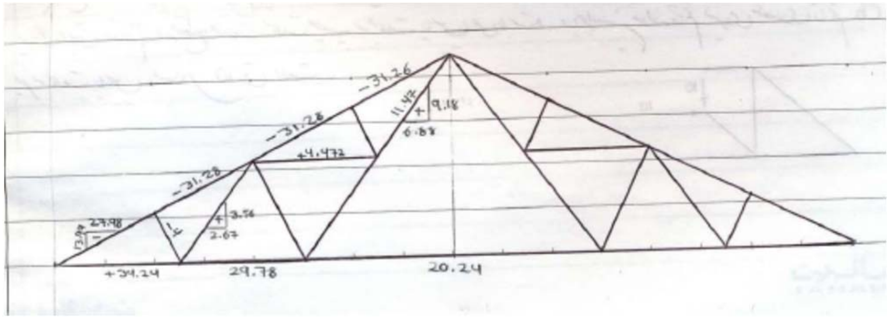
مثال: خرپای مقابل را تحلیل کنید

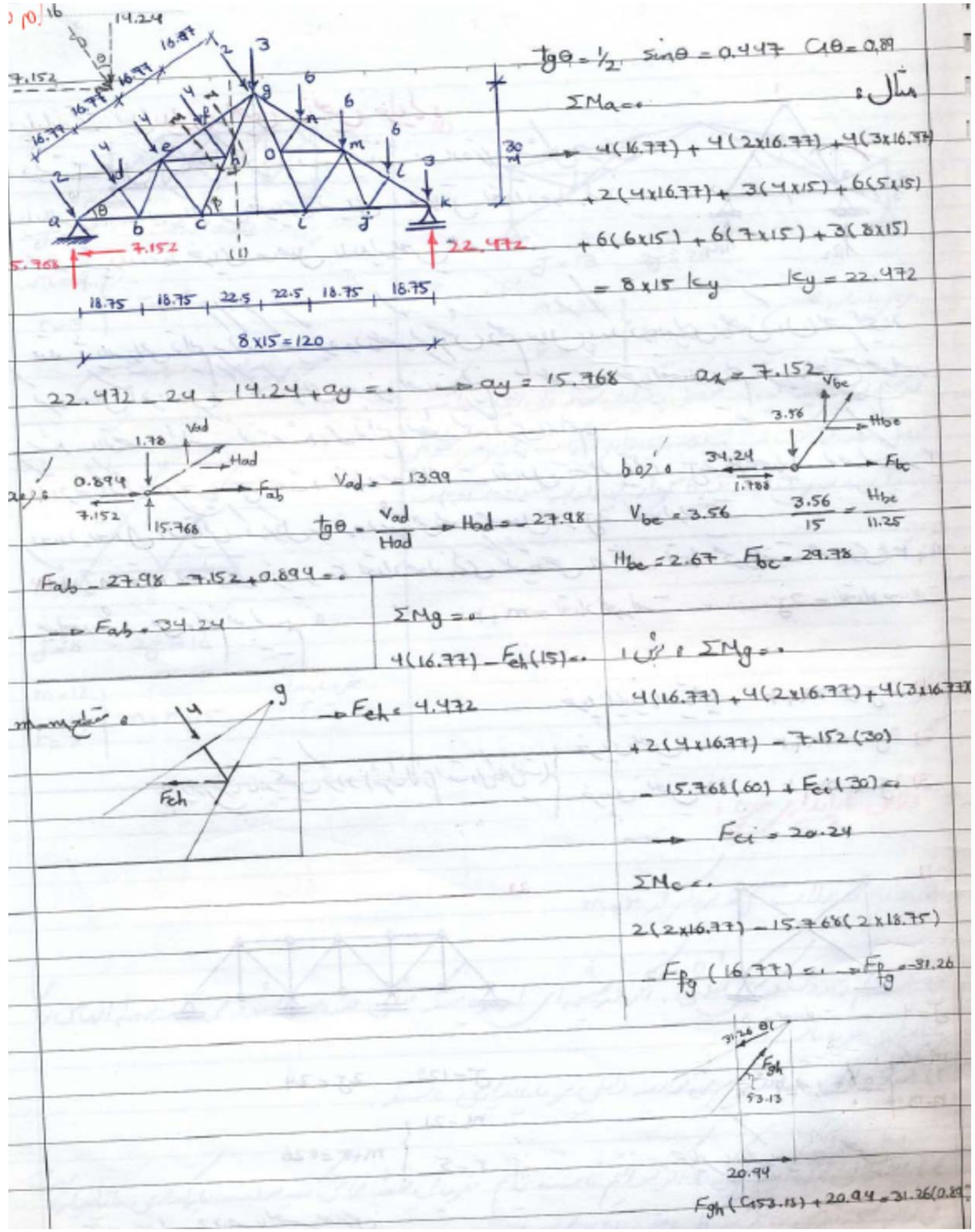
$$F_{de} = \frac{2643.8}{13.8} = 191.6$$

$$H_{DE} = -\frac{2745}{15} = -183 \text{ ton}$$

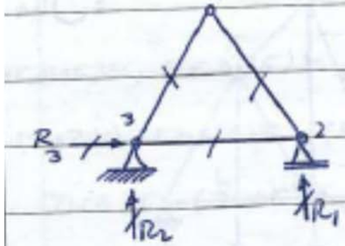
$$V_{DE} = -24.4$$




-31.26
-31.28
-31.28
11.42
9.18
6.88
+4.472
27.98
13.99
3.56
2.67
+34.24
29.78
20.24


tgθ = 1/2   sinθ = 0.447   Cosθ = 0.89
مثال:
ΣMa = 0
4(16.77) + 4(2x16.77) + 4(3x16.77)
+ 2(4x16.77) + 3(4x15) + 6(5x15)
+ 6(6x15) + 6(7x15) + 3(8x15)
= 8x15 ky   ky = 22.472
8x15 = 120
22.472 - 24 - 14.24 + ay = 0   ay = 15.768   ax = 7.152
Vad = -13.99
tgθ = Vad/Had   Had = -27.98
Fab - 27.98 - 7.152 + 0.894 = 0
Fab = 34.24
Vbe = 3.56   3.56/15 = Hbe/11.25
Hbe = 2.67   Fbc = 29.78
ΣMg = 0
4(16.77) - Feh(15) = 0
Feh = 4.472
ΣMg = 0
4(16.77) + 4(2x16.77) + 4(3x16.77)
+ 2(4x16.77) - 7.152(30)
- 15.768(60) + Fci(30) = 0
Fci = 20.24
ΣMc = 0
2(2x16.77) - 15.768(2x18.75)
- Ffg(16.77) = 0   Ffg = -31.26
Fgh(Cos53.13) + 20.94 = 31.26(0.89)

پایداری و ناپایداری، معینی و نامعینی خرپاها



در شکل مقابل مشاهده می گردد یک مثلث ساده ترین شکل خرپایی پایدار می باشد. اگر این خرپای مبنا با حداقل تعداد مولفه های تکیه گاهی تکیه داده شود خرپای حاصل پایدار و معین است.

مثلث پایه را می توانیم به کمک یک گره و دو عضو گسترش دهیم بدون آنکه در پایداری و معینی آن تغییری ایجاد گردد. حال اگر بدون آنکه گره ای به خرپا اضافه کنیم، عضوی به خرپا اضافه نماییم خرپا نامعین خواهد شد. با توجه به مطالب گفته شده می توانیم این طور نتیجه گیری نماییم:

۱) اگر تعداد گره های خرپا J باشد تعداد معادلات تعادل استاتیکی 2J خواهد بود. توجه شود که ۳ معادله تعادل استاتیکی که برای کل سازه نوشته می شود مستقل از 2J معادله نیست.

۲) اگر m تعداد اعضای خرپا و r تعداد مولفه های تکیه گاهی باشد، تعداد مجهولات مساوی m+r خواهد شد. می توانیم بنویسیم:

تعداد معادلات $= 2J$

تعداد مجهولات $= m + r$

1) $2J > m + r$ خرپا ناپایدار استاتیکی

2) $2J = m + r$ خرپای معین

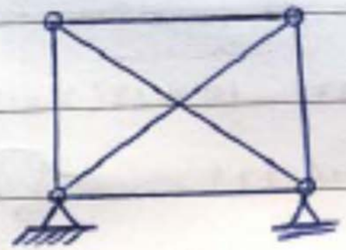
3) $2J < m + r$ خرپای نامعین

(در حالت‌های ۲ و ۳:) پایداری خرپا باید تحقیق گردد (شرط لازم است ولی کافی نیست)

$r < 3$	the structure is statically unstable externally
$r = 3$	the structure is statically determinate externally
$r > 3$	the structure is statically indeterminate externally

$r < 3$	the structure is statically unstable externally
$r = 3$	the structure is statically determinate externally
$r > 3$	the structure is statically indeterminate externally

1)

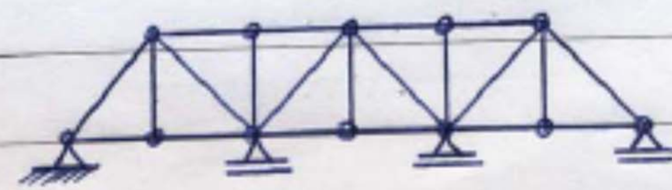


$J=4$ → معادلات $=8$

$m=6$, $r=3$ } → مجهولات $=9$

خرپایی پایدار و یک درجه نامعین

2)



$J=12$ $2J=24$

$m=21$, $r=5$ } $m+r=26$

خرپای پایدار و 2 درجه نامعین خارجی

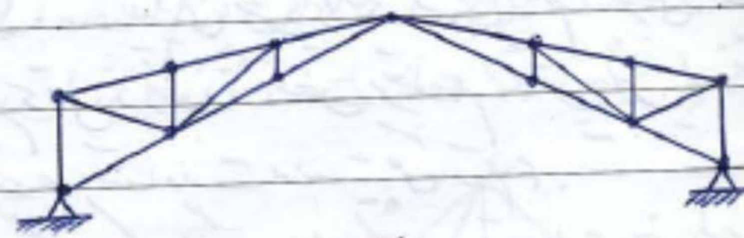
3)

$J=10$ معادلات $=20$

$m=19$, $r=3$ } مجهولات $=22$

خرپای پایدار و 2 درجه نامعین

4)



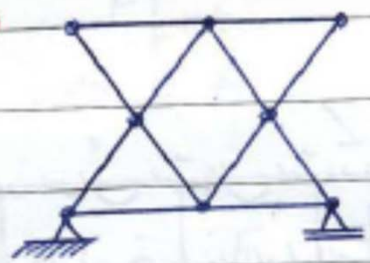
$J=13$ $2J=26$

$m=22$, $r=4$ } $m+r=26$

خرپای پایدار و معین

دو جسم توسط یک میله و یک مفصل پایداری شدند ولی این نقطه یک مفصل بکار رفته که باعث یک درجه ناپایداری گشته است. برای جبران این ناپایداری به جای 3 مولفه تکیه گاهی از چهار مولفه استفاده شده است.

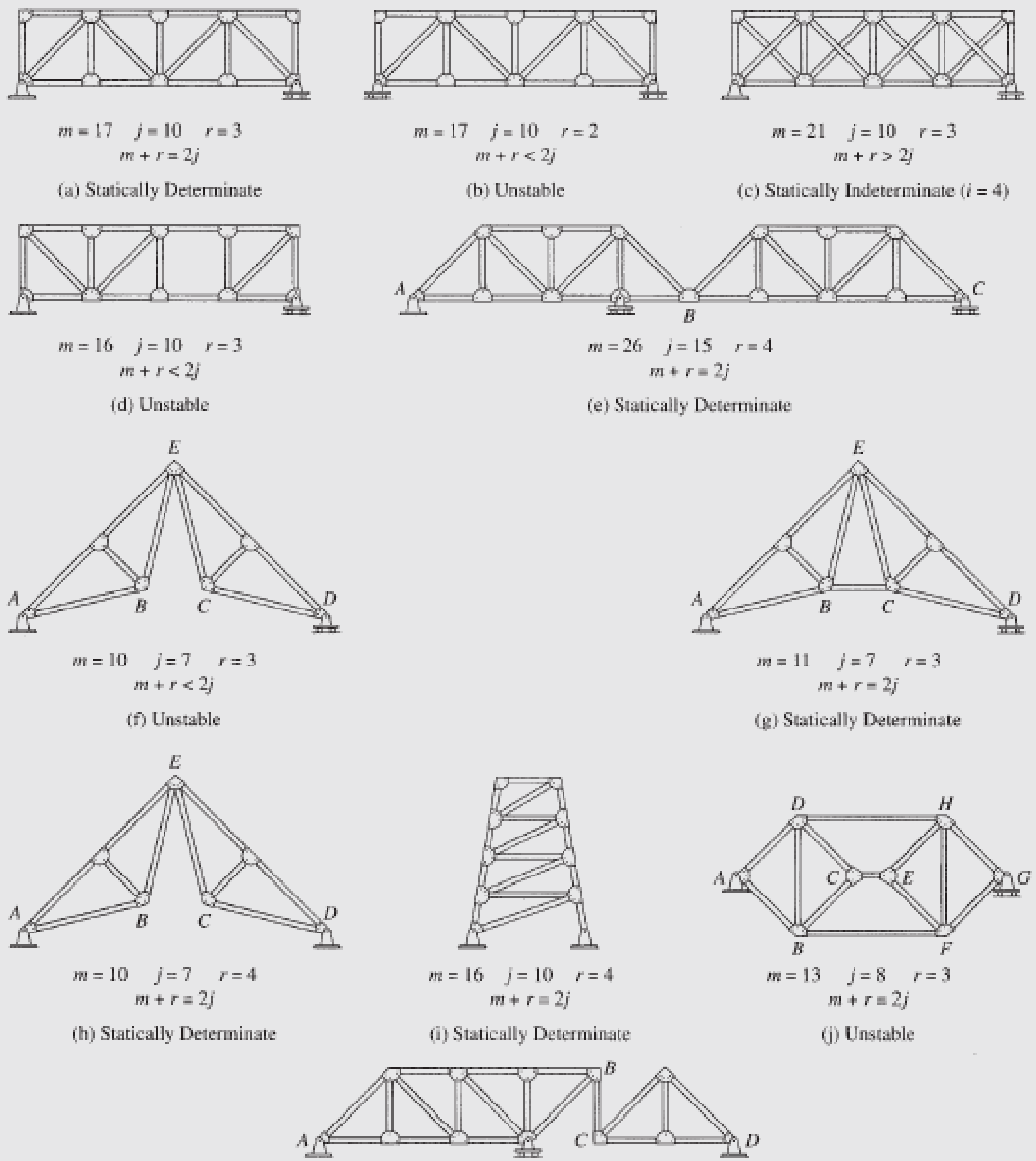
5)



$J=8$ $2J=16$

$m=12$, $r=3$ } $m+r=15$

خرپا ناپایدار است.



$m = 17 \quad j = 10 \quad r = 3$
$m + r = 2j$
(a) Statically Determinate

$m = 17 \quad j = 10 \quad r = 2$
$m + r < 2j$
(b) Unstable

$m = 21 \quad j = 10 \quad r = 3$
$m + r > 2j$
(c) Statically Indeterminate ($i = 4$)

$m = 16 \quad j = 10 \quad r = 3$
$m + r < 2j$
(d) Unstable

$m = 26 \quad j = 15 \quad r = 4$
$m + r = 2j$
(e) Statically Determinate

$m = 10 \quad j = 7 \quad r = 3$
$m + r < 2j$
(f) Unstable

$m = 11 \quad j = 7 \quad r = 3$
$m + r = 2j$
(g) Statically Determinate

$m = 10 \quad j = 7 \quad r = 4$
$m + r = 2j$
(h) Statically Determinate

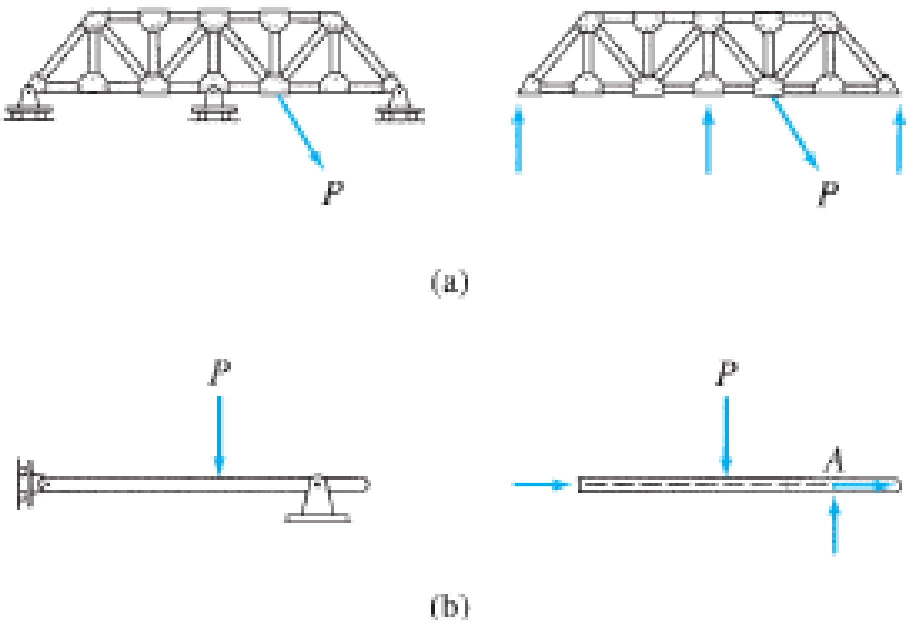
$m = 16 \quad j = 10 \quad r = 4$
$m + r = 2j$
(i) Statically Determinate

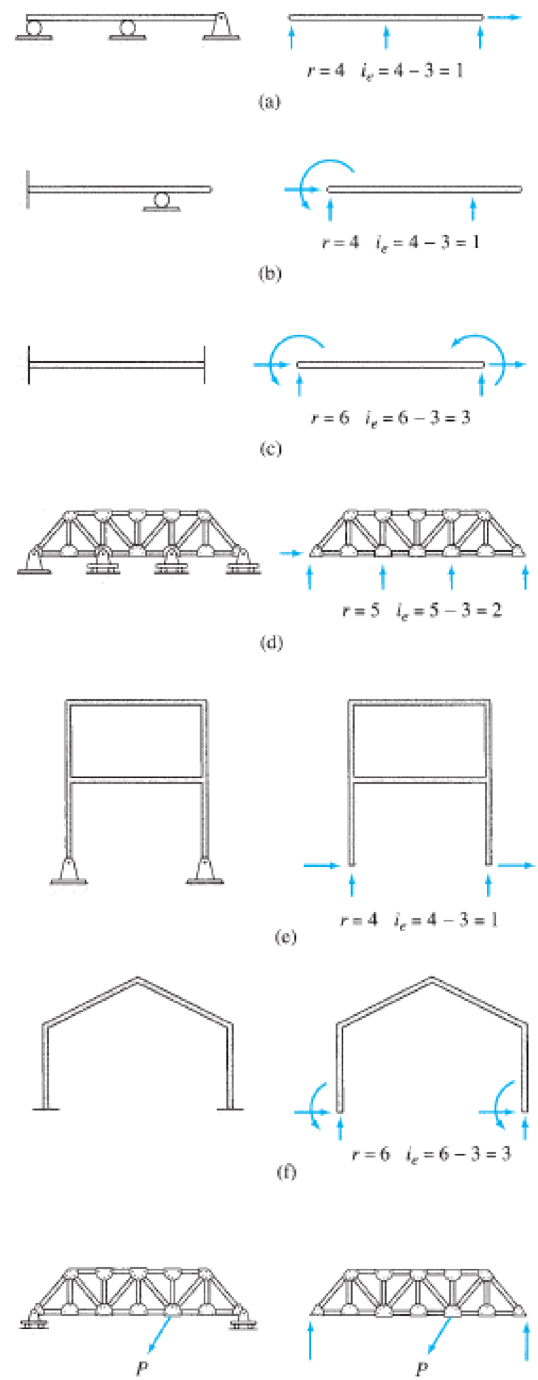
$m = 13 \quad j = 8 \quad r = 3$
$m + r = 2j$
(j) Unstable

$m = 19 \quad j = 12 \quad r = 5$
$m + r = 2j$
(k) Statically Determinate

FIG. 4.15

continued


P
P
(a)
P
P
A
(b)


r = 4   i_e = 4 − 3 = 1
(a)
r = 4   i_e = 4 − 3 = 1
(b)
r = 6   i_e = 6 − 3 = 3
(c)
r = 5   i_e = 5 − 3 = 2
(d)
r = 4   i_e = 4 − 3 = 1
(e)
r = 6   i_e = 6 − 3 = 3
(f)
P
P

For an externally indeterminate structure, the degree of external indeterminacy is expressed as

$$i_e = r - (3 + e_c) \tag{3.10}$$

Alternative Approach An alternative approach that can be used for determining the static instability, determinacy, and indeterminacy of internally unstable structures is as follows:

1. Count the total number of support reactions, r.
2. Count the total number of internal forces, f_i, that can be transmitted through the internal hinges and the internal rollers of the structure. Recall that an internal hinge can transmit two force components, and an internal roller can transmit one force component.
3. Determine the total number of unknowns, $r + f_i$.
4. Count the number of rigid members or portions, n_r, contained in the structure.
5. Because each of the individual rigid portions or members of the structure must be in equilibrium under the action of applied loads, reactions, and/or internal forces, each member must satisfy the three equations of equilibrium ($\sum F_x = 0$, $\sum F_y = 0$, and $\sum M = 0$). Thus, the total number of equations available for the entire structure is $3n_r$.
6. Determine whether the structure is statically unstable, determinate, or indeterminate by comparing the total number of unknowns, $r + f_i$, to the total number of equations. If

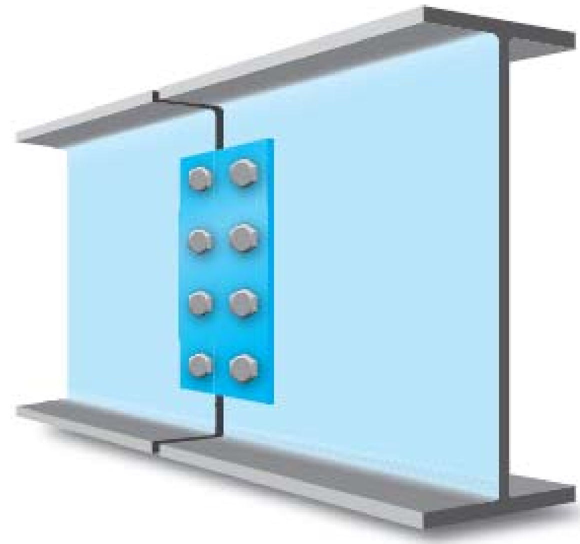
$$\begin{aligned} r + f_i &< 3n_r && \text{the structure is statically unstable externally} \\ r + f_i &= 3n_r && \text{the structure is statically determinate externally} \\ r + f_i &> 3n_r && \text{the structure is statically indeterminate externally} \end{aligned} \tag{3.11}$$

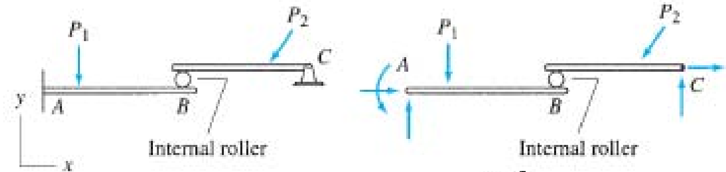
For indeterminate structures, the degree of external indeterminacy is given by

$$i_e = (r + f_i) - 3n_r \tag{3.12}$$

Applying this alternative procedure to the structure of Fig. 3.13(b), we can see that for this structure, $r = 4$, $f_i = 2$, and $n_r = 2$. As the total number of unknowns ($r + f_i = 6$) is equal to the total number of equations ($3n_r = 6$), the structure is statically determinate externally. Similarly, for the structure of Fig. 3.15, $r = 5$, $f_i = 1$, and $n_r = 2$. Since $r + f_i = 3n_r$, this structure is also statically determinate externally.

The criteria for the static determinacy and indeterminacy as described in Eqs. (3.9) and (3.11), although necessary, are not sufficient because they cannot account for the possibility of geometric instability. To avoid geometric instability, the internally unstable structures, like the internally stable structures considered previously, must be supported by





$r = 5$

Two equations of condition: $\Sigma F_x^{AB} = 0$ or $\Sigma F_x^{BC} = 0$

$\Sigma M_B^{AB} = 0$ or $\Sigma M_B^{BC} = 0$

and

$$\sum M_B^{AB} = 0 \quad \text{or} \quad \sum M_B^{BC} = 0$$

These two equations of condition can be used in conjunction with the three equilibrium equations to determine the five unknown external reactions. Thus, the structure of Fig. 3.15 is statically determinate externally.

From the foregoing discussion, we can conclude that if there are e_c equations of condition (one equation for each internal hinge and two equations for each internal roller) for an internally unstable structure, which is supported by r external reactions, then if

$$\begin{aligned} r &< 3 + e_c && \text{the structure is statically unstable externally} \\ r &= 3 + e_c && \text{the structure is statically determinate externally} \\ r &> 3 + e_c && \text{the structure is statically indeterminate externally} \end{aligned} \tag{3.9}$$

Example 3.1

Classify each of the structures shown in Fig. 3.17 as externally unstable, statically determinate, or statically indeterminate. If the structure is statically indeterminate externally, then determine the degree of external indeterminacy.

Solution

(a) This beam is internally stable with $r = 5 > 3$. Therefore, it is statically indeterminate externally with the degree of external indeterminacy of

$$i_e = r - 3 = 5 - 3 = 2 \qquad \textbf{Ans.}$$

(b) This beam is internally unstable. It is composed of two rigid members AB and BC connected by an internal hinge at B. For this beam, $r = 6$ and $e_c = 1$. Since $r > 3 + e_c$, the structure is statically indeterminate externally with the degree of external indeterminacy of

$$i_e = r - (3 + e_c) = 6 - (3 + 1) = 2 \qquad \textbf{Ans.}$$

Alternative Method. $f_i = 2$, $n_r = 2$, $r + f_i = 6 + 2 = 8$, and $3n_r = 3(2) = 6$. As $r + f_i > 3n_r$, the beam is statically indeterminate externally, with

$$i_e = (r + f_i) - 3n_r = 8 - 6 = 2 \qquad \textbf{Checks}$$

(c) This structure is internally unstable with $r = 4$ and $e_c = 2$. Since $r < 3 + e_c$, the structure is statically unstable externally. This can be verified from the figure, which shows that the member BC is not restrained against movement in the horizontal direction. **Ans.**

Alternative Method. $f_i = 1$, $n_r = 2$, $r + f_i = 4 + 1 = 5$, and $3n_r = 6$. Since $r + f_i < 3n_r$, the structure is statically unstable externally. **Checks**

(d) This beam is internally unstable with $r = 5$ and $e_c = 2$. Because $r = 3 + e_c$, the beam is statically determinate externally. **Ans.**

continued

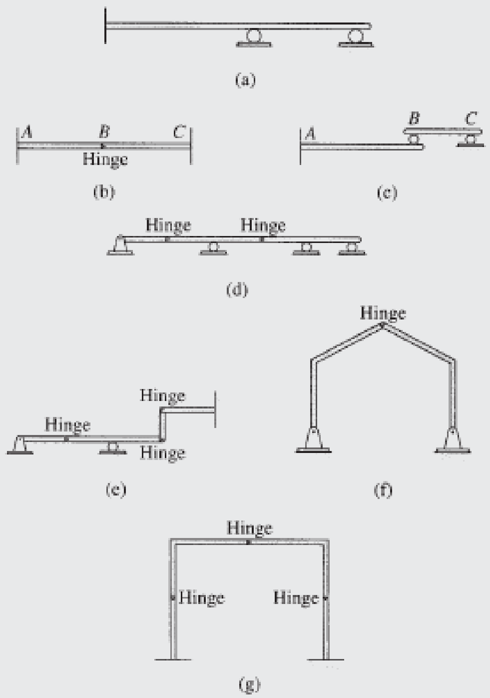


FIG. 3.17

Alternative Method. $f_i = 4$, $n_r = 3$, $r + f_i = 5 + 4 = 9$, and $3n_r = 3(3) = 9$. Because $r + f_i = 3n_r$, the beam is staticaly determinate externally. **Checks**

(e) This is an internally unstable structure with $r = 6$ and $e_c = 3$. Since $r = 3 + e_c$, the structure is statically determinate externally. **Ans.**

Alternative Method. $f_i = 6$, $n_r = 4$, $r + f_i = 6 + 6 = 12$, and $3n_r = 3(4) = 12$. Because $r + f_i = 3n_r$, the structure is statically determinate externally. **Checks**

(f) This frame is internally unstable with $r = 4$ and $e_c = 1$. Since $r = 3 + e_c$, the frame is statically determinate externally. **Ans.**

Alternative Method. $f_i = 2$, $n_r = 2$, $r + f_i = 4 + 2 = 6$, and $3n_r = 3(2) = 6$. Since $r + f_i = 3n_r$, the frame is statically determinate externally. **Checks**

(g) This frame is internally unstable with $r = 6$ and $e_c = 3$. Since $r = 3 + e_c$, the frame is statically determinate externally. **Ans.**

Alternative Method. $f_i = 6$, $n_r = 4$, $r + f_i = 6 + 6 = 12$, and $3n_r = 3(4) = 12$. Because $r + f_i = 3n_r$, the frame is statically determinate externally. **Checks**

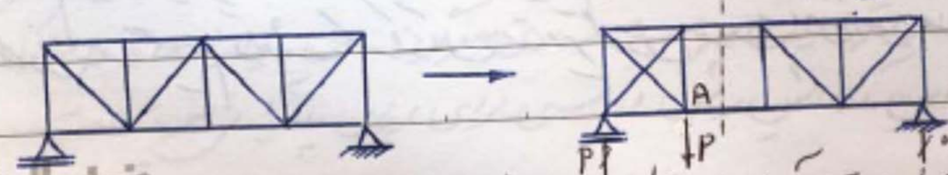
دلایل ناپایداری خرپا ،

۱) تعداد معادلات $2J$ کمتر از $m+r$.

۲) عدم وجود شکل‌های مثلثی. اگر قسمتی از خرپا شکل مثلثی مشخص نشود می‌تواند نقطه ابهامی برای ناپایداری خرپا باشد.

۳) تکیه‌گاه‌های ناپایدار نیز می‌توانند دلیلی بر ناپایداری باشند.

۴) استدلال‌های استاتیکی ؛ اگر بتوانیم ثابت نماییم خرپایی فقط برای یک حالت بارگذاری ناپایدار است می‌توانیم نتیجه‌گیری نماییم که این خرپا ناپایدار است.



J ، m ، r تغییری نکرد.

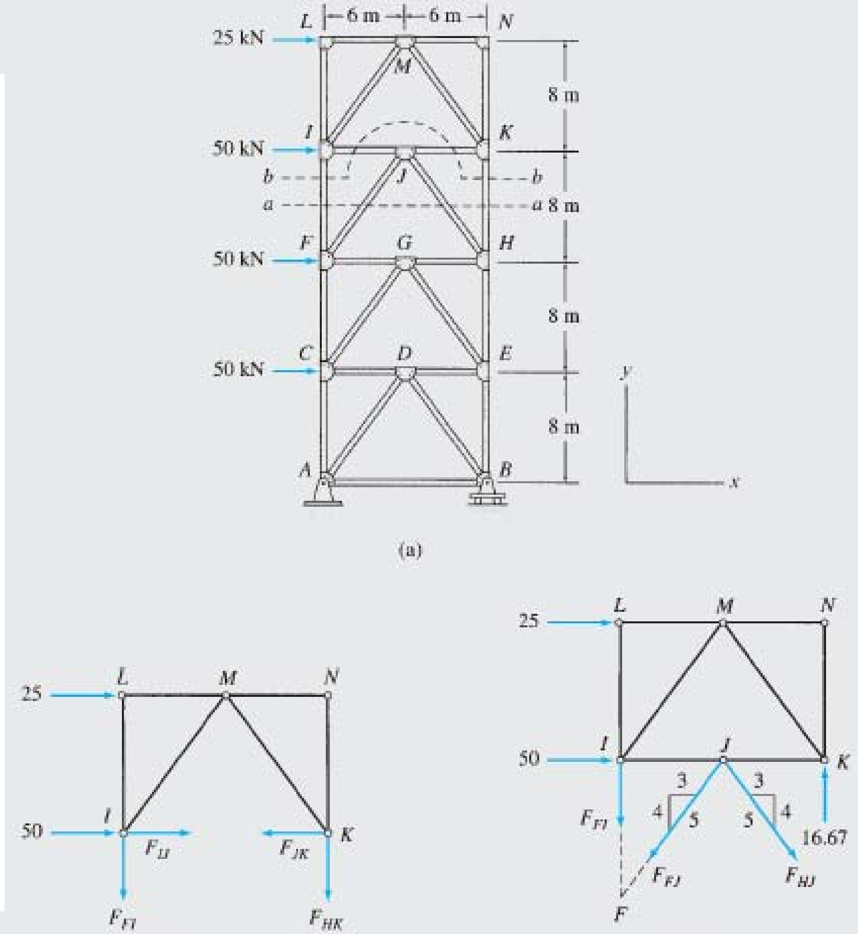
بار P را به A وارد می‌کنیم. تکیه‌گاه‌ها را بدست می‌آوریم. (با مقطع زدن در عضو راست A) سپس $\Sigma M=0$ برقرار نمی‌باشد.

۵

۵) تحلیل خرپا. اگر نتوانیم با یکی از روش‌های گفته شده ناپایداری خرپا را نتیجه‌گیری نماییم، برای تشخیص ناپایداری اقدام به تحلیل خرپا می‌نماییم. اگر در حین تحلیل به وجود تناقض برخوردیم، معنی‌اش این است که خرپا ناپایدار می‌باشد. (نیروی عضوی را از دو راه دو عدد متفاوت بدست می‌آوریم).

Example 4.9

Determine the forces in members FJ, HJ, and HK of the K truss shown in Fig. 4.24(a) by the method of sections.



(a)

(b) Section bb

(c) Section aa

FIG. 4.24

Solution

From Fig. 4.24(a), we can observe that the horizontal section aa passing through the three members of interest, FJ, HJ, and HK, also cuts an additional member FI, thereby releasing four unknowns, which cannot be determined by three equations of equilibrium. Trusses such as the one being considered here with the members arranged in the form of the letter K can be analyzed by a section curved around the middle joint, like section bb shown in Fig. 4.24(a). To avoid the calculation of support reactions, we will use the upper portion $IKNL$ of the truss above section bb for analysis. The free-body diagram of this portion is shown in Fig. 4.24(b). It can be seen that although section bb has cut four members, FI, IJ, JK, and HK, forces in members FI and HK can be determined by summing moments about points K and I,

continued

respectively, because the lines of action of three of the four unknowns pass through these points. We will, therefore, first compute F_{HK} by considering section bb and then use section aa to determine F_{FJ} and F_{HJ}.

Section *bb*. Using Fig. 4.24(b), we write

$$+\circlearrowleft \sum M_I = 0 \qquad -25(8) - F_{HK}(12) = 0$$

$$F_{HK} = -16.67 \text{ kN}$$

$$F_{HK} = 16.67 \text{ kN (C)} \qquad \textbf{Ans.}$$

Section *aa*. The free-body diagram of the portion $IKNL$ of the truss above section aa is shown in Fig. 4.24(c). To determine F_{HJ}, we sum moments about F, which is the point of intersection of the lines of action of F_{FI} and F_{FJ}. Thus,

$$+\circlearrowleft \sum M_F = 0 \qquad -25(16) - 50(8) + 16.67(12) - \frac{3}{5}F_{HJ}(8) - \frac{4}{5}F_{HJ}(6) = 0$$

$$F_{HJ} = -62.5 \text{ kN}$$

$$F_{HJ} = 62.5 \text{ kN (C)} \qquad \textbf{Ans.}$$

By summing forces in the horizontal direction, we obtain

$$+\rightarrow \sum F_x = 0 \qquad 25 + 50 - \frac{3}{5}F_{FJ} - \frac{3}{5}(62.5) = 0$$

$$F_{FJ} = 62.5 \text{ kN (T)} \qquad \textbf{Ans.}$$

Checking Computations. Finally, to check our calculations, we apply an alternative equilibrium equation, which involves the three member forces determined by the analysis. Using Fig. 4.24(c), we write

$$+\circlearrowleft \sum M_I = -25(8) - \frac{4}{5}(62.5)(6) + \frac{4}{5}(62.5)(6) + 16.67(12) = 0 \qquad \textbf{Checks}$$

Solution

Static Determinacy. The truss has 11 members and 7 joints and is supported by 3 reactions. Since $m + r = 2j$ and the reactions and the members of the truss are properly arranged, it is statically determinate.

The slopes of the inclined members, as determined from the dimensions of the truss, are shown in Fig. 4.25(a).

Reactions. The reactions at supports A and B, as computed by applying the three equilibrium equations to the free-body diagram of the entire truss (Fig. 4.25(b)), are

$$A_x = 25 \text{ k} \leftarrow \qquad A_y = 5 \text{ k} \uparrow \qquad B_y = 35 \text{ k} \uparrow$$

Section *aa*. Since a joint with two or fewer unknown forces cannot be found to start the method of joints, we first calculate F_{AB} by using section aa, as shown in Fig. 4.25(a).

The free-body diagram of the portion of the truss on the left side of section aa is shown in Fig. 4.25(c). We determine F_{AB} by summing moments about point G, the point of intersection of the lines of action of F_{CG} and F_{DG}.

$$+\circlearrowleft \sum M_G = 0 \qquad -25(32) - 5(16) + 10(16) + F_{AB}(32) = 0$$

$$F_{AB} = 22.5 \text{ k (T)} \qquad \textbf{Ans.}$$

With F_{AB} now known, the method of joints can be started either at joint A, or at joint B, since both of these joints have only two unknowns each. We begin with joint A.

Joint *A*. The free-body diagram of joint A is shown in Fig. 4.25(d).

$$+\rightarrow \sum F_x = 0 \qquad -25 + 22.5 + \frac{1}{\sqrt{5}}F_{AC} + \frac{3}{5}F_{AD} = 0$$

$$+\uparrow \sum F_y = 0 \qquad 5 + \frac{2}{\sqrt{5}}F_{AC} + \frac{4}{5}F_{AD} = 0$$

Solving these equations simultaneously, we obtain

$$F_{AC} = -27.95 \text{ k} \quad \text{and} \quad F_{AD} = 25 \text{ k}$$

$$F_{AC} = 27.95 \text{ k (C)} \qquad \textbf{Ans.}$$

$$F_{AD} = 25 \text{ k (T)} \qquad \textbf{Ans.}$$

Joints *C* and *D*. Focusing our attention on joints C and D in Fig. 4.25(b), and by satisfying the two equilibrium equations by inspection at each of these joints, we determine

$$F_{CG} = 27.95 \text{ k (C)} \qquad \textbf{Ans.}$$

$$F_{CD} = 10 \text{ k (C)} \qquad \textbf{Ans.}$$

$$F_{DG} = 20.62 \text{ k (T)} \qquad \textbf{Ans.}$$

Joint *G*. Next, we consider the equilibrium of joint G (see Fig. 4.25(e)).

$$+\rightarrow \sum F_x = 0 \qquad 5 + \frac{1}{\sqrt{5}}(27.95) - \frac{1}{\sqrt{17}}(20.62) + \frac{1}{\sqrt{17}}F_{EG} + \frac{1}{\sqrt{5}}F_{FG} = 0$$

$$+\uparrow \sum F_y = 0 \qquad -40 + \frac{2}{\sqrt{5}}(27.95) - \frac{4}{\sqrt{17}}(20.62) - \frac{4}{\sqrt{17}}F_{EG} - \frac{2}{\sqrt{5}}F_{FG} = 0$$

continued

Example 4.10

Determine the force in each member of the compound truss shown in Fig. 4.25(a).

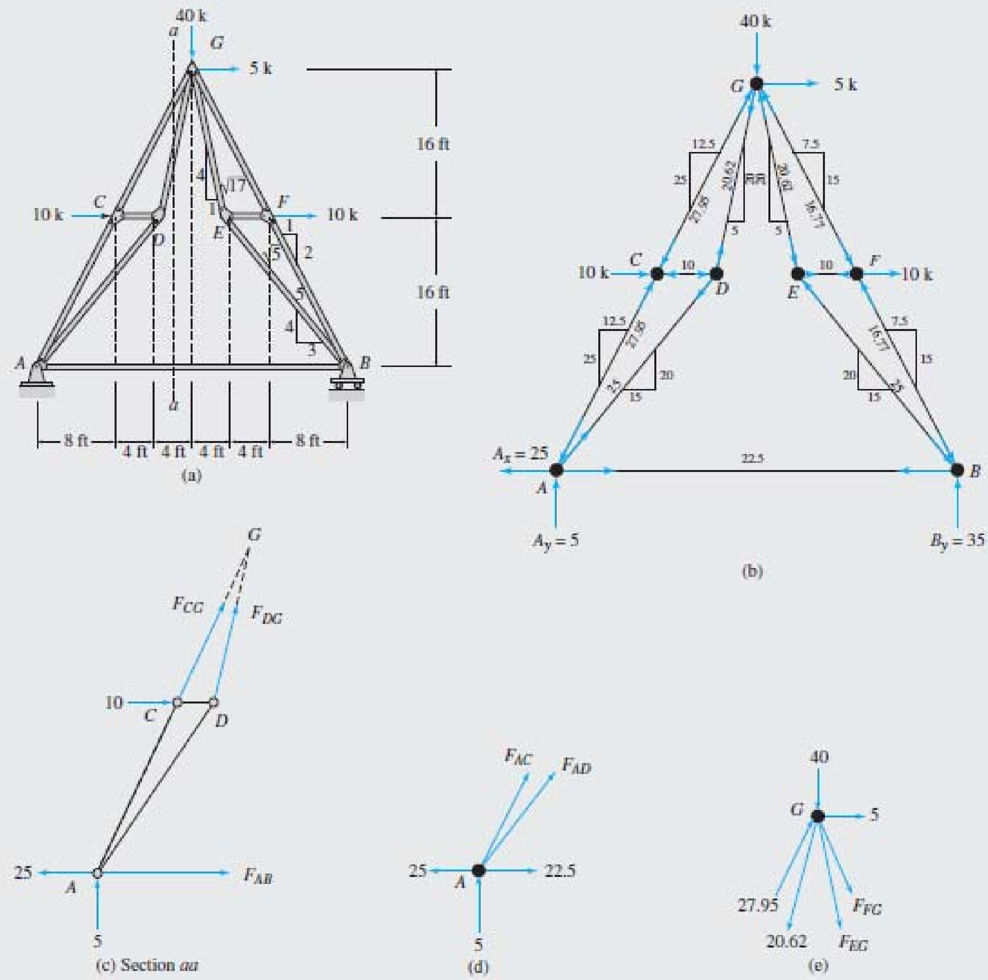


FIG. 4.25

continued

Solving these equations, we obtain

$$F_{EG} = -20.62 \text{ k} \quad \text{and} \quad F_{FG} = -16.77 \text{ k}$$

$$F_{EG} = 20.62 \text{ k (C)} \qquad \textbf{Ans.}$$

$$F_{FG} = 16.77 \text{ k (C)} \qquad \textbf{Ans.}$$

Joints *E* and *F*. Finally, by considering the equilibrium, by inspection, of joints E and F (see Fig. 4.25(b)), we obtain

$$F_{BE} = 25 \text{ k (C)} \qquad \textbf{Ans.}$$

$$F_{EF} = 10 \text{ k (T)} \qquad \textbf{Ans.}$$

$$F_{BF} = 16.77 \text{ k (C)} \qquad \textbf{Ans.}$$

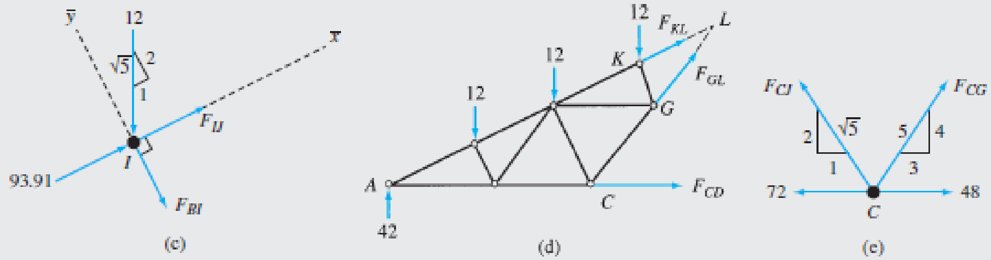


FIG. 4.26 (contd.)

Solution

The Fink truss shown in Fig. 4.26(a) is a compound truss formed by connecting two simple trusses, ACL and DFL, by a common joint L and a member CD.

Static Determinacy. The truss contains 27 members and 15 joints and is supported by 3 reactions. Because $m + r = 2j$ and the reactions and the members of the truss are properly arranged, it is statically determinate.

Reactions. The reactions at supports A and F of the truss, as computed by applying the three equations of equilibrium to the free-body diagram of the entire truss (Fig. 4.26(b)), are

$$A_x = 0 \qquad A_y = 42 \text{ k}\uparrow \qquad F_y = 42 \text{ k}\uparrow$$

Joint *A*. The method of joints can now be started at joint A, which has only two unknown forces, F_{AB} and F_{AI}, acting on it. By inspection of the forces acting at this joint (see Fig. 4.26(b)), we obtain the following:

$$F_{AI} = 93.91 \text{ k (C)}$$ **Ans.**

$$F_{AB} = 84 \text{ k (T)}$$ **Ans.**

Joint *I*. The free-body diagram of joint I is shown in Fig. 4.26(c). Member BI is perpendicular to members AI and IJ, which are collinear, so the computation of member forces can be simplified by using an $\bar{x}$ axis in the direction of the collinear members, as shown in Fig. 4.26(c).

$$+\nwarrow \sum F_{\bar{y}} = 0 \qquad -\frac{2}{\sqrt{5}}(12) - F_{BI} = 0$$

$$F_{BI} = -10.73 \text{ k}$$

$$F_{BI} = 10.73 \text{ k (C)}$$ **Ans.**

$$+\nearrow \sum F_{\bar{x}} = 0 \qquad 93.91 - \frac{1}{\sqrt{5}}(12) + F_{IJ} = 0$$

$$F_{IJ} = -88.54 \text{ k}$$

$$F_{IJ} = 88.54 \text{ k (C)}$$ **Ans.**

Joint *B*. Considering the equilibrium of joint B, we obtain (see Fig. 4.26(b)) the following:

$$+\uparrow \sum F_y = 0 \qquad -\frac{2}{\sqrt{5}}(10.73) + \frac{4}{5}F_{BJ} = 0$$

$$F_{BJ} = 12 \text{ k (T)}$$ **Ans.**

$$+\rightarrow \sum F_x = 0 \qquad -84 + \frac{1}{\sqrt{5}}(10.73) + \frac{3}{5}(12) + F_{BC} = 0$$

$$F_{BC} = 72 \text{ k (T)}$$ **Ans.**

continued

Example 4.11

Determine the force in each member of the Fink truss shown in Fig. 4.26(a).

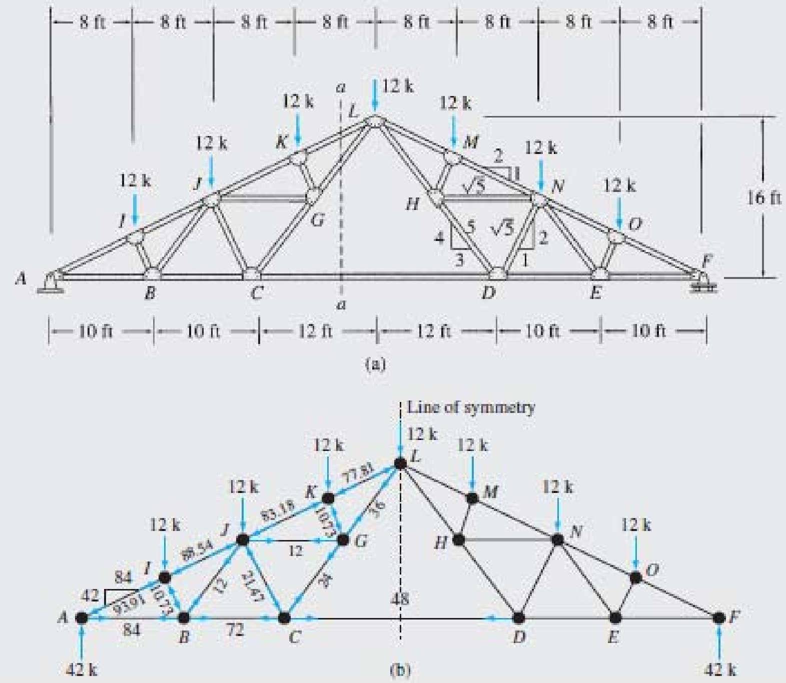


FIG. 4.26

continued

Section *aa*. Since at each of the next two joints, C and J, there are three unknowns (F_{CD}, F_{CG}, and F_{CJ} at joint C and F_{CJ}, F_{GJ}, and F_{JK} at joint J), we calculate F_{CD} by using section *aa*, as shown in Fig. 4.26(a). (If we moved to joint F and started computing member forces from that end of the truss, we would encounter similar difficulties at joints D and N.)

The free-body diagram of the portion of the truss on the left side of section *aa* is shown in Fig. 4.26(d). We determine F_{CD} by summing moments about point L, the point of intersection of the lines of action of F_{GL} and F_{KL}.

$$+\circlearrowleft \sum M_L = 0 \qquad -42(32) + 12(24) + 12(16) + 12(8) + F_{CD}(16) = 0$$

$$F_{CD} = 48 \text{ k (T)} \qquad \textbf{Ans.}$$

Joint C. With F_{CD} now known, there are only two unknowns, F_{CG} and F_{CJ}, at joint C. These forces can be determined by applying the two equations of equilibrium to the free body of joint C, as shown in Fig. 4.26(e).

$$+\uparrow \sum F_y = 0 \qquad \frac{2}{\sqrt{5}}F_{CJ} + \frac{4}{5}F_{CG} = 0$$

$$+\rightarrow \sum F_x = 0 \qquad -72 + 48 - \frac{1}{\sqrt{5}}F_{CJ} + \frac{3}{5}F_{CG} = 0$$

Solving these equations simultaneously, we obtain

$$F_{CJ} = -21.47 \text{ k} \quad \text{and} \quad F_{CG} = 24 \text{ k}$$

$$F_{CJ} = 21.47 \text{ k (C)} \qquad \textbf{Ans.}$$

$$F_{CG} = 24 \text{ k (T)} \qquad \textbf{Ans.}$$

Joints J, K, and G. Similarly, by successively considering the equilibrium of joints J, K, and G, in that order, we determine the following:

$$F_{JK} = 83.18 \text{ k (C)} \qquad \textbf{Ans.}$$

$$F_{GJ} = 12 \text{ k (T)} \qquad \textbf{Ans.}$$

$$F_{KL} = 77.81 \text{ k (C)} \qquad \textbf{Ans.}$$

$$F_{GK} = 10.73 \text{ k (C)} \qquad \textbf{Ans.}$$

$$F_{GL} = 36 \text{ k (T)} \qquad \textbf{Ans.}$$

Symmetry. Since the geometry of the truss and the applied loading are symmetrical about the center line of the truss (shown in Fig. 4.26(b)), its member forces will also be symmetrical with respect to the line of symmetry. It is, therefore, sufficient to determine member forces in only one-half of the truss. The member forces determined here for the left half of the truss are shown in Fig. 4.26(b). The forces in the right half can be obtained from the consideration of symmetry; for example, the force in member MN is equal to that in member JK, and so forth. The reader is urged to verify this by computing a few member forces in the right half of the truss. **Ans.**